IMAGES NZ

ISSUE ELEVEN

The maths/art nexus

Until the end of 2010, expatriate New Zealander Peter James Smith was Professor of Mathematics and Art at RMIT in Victoria, Australia.

Smith has straddled two very different fields for three decades. He has written an influential statistics textbook, and exhibited his landscape oils covered with handwritten maths notation, as well as work in other media in regular solo exhibitions since the 1970s.

He first mixed mathematics and art in an exhibition of paintings using the Fibonacci numbers in Auckland in 1977. "People encourage trans-disciplinary work," he says, "but it's very difficult to assess. It's important to do because the education system separates people from age 12 into one stream or the other. It's a very destructive split. Often teams of people collaborate across art and science disciplines, but they don't understand what the other side has done - it's not seamless."

He concentrates on the language of maths in his paintings. "I don't like beautiful symmetric diagrams; the formulae that produce symmetry are interesting but not the picture of symmetry. It's wonderful to bring to non-mathematicians some of the simple delights of how group theory works - the 1+1= 0 argument - they're so used to the language that they don't know what a gift it is."

The mathematical language in his paintings has included data sets, such as 1880s experiments on speed of light, and the orbital elements (position, location and appearance) of Halley's Comet. "It illustrates how statistics is such a powerful thing - by gathering that data you get to know about the world. I used a lot of theorems and simple proofs, often from number theory, and my own research; I produced a new result on a painting before it was published in a journal. That's when the nexus is working really well, having that moment of insight when you're working on a canvas that mathematicians have at the blackboard."

He was in New Zealand to give a lecture on Truth + Beauty, the title of his recent solo exhibitions and of a book he is writing. "It contextualises the mark making on my paintings over the last 30 years. It will have a lot of maths - understanding the nature of proof, deductive reasoning; all those things art people don't know. The process of proofs and the failed alleyways that mathematicians go down to discover things are very precious."

"Starting with axioms and definitions and constructing and proving theorems - just like Euclid built all geometry from five axioms - that's the wonder and the magic."

Smith was one of two Antarctic New Zealand Artist Fellows in early 2010. Their most mathematically interesting find was NASA's website tracking of icebergs between 2000 and 2005, when they drifted north of Christchurch. "The traces they left bordered on chaos theory; it's an interesting relationship between something mathematically chaotic and some scribbled mark."

In 2010, Smith was applying linear regression to art and real estate markets. "It's ironic that when you retire, your research suddenly starts looking hopeful! I was working with a database of realised secondary market (auction) prices for art, which is an example of left censoring. The information you have is not the actual realised price, but you know the price is less than the reserve because it was passed in at auction. The reserve therefore becomes a left censored data point. Real estate people would pay millions to type ▶2

Top: *Ice Station*, 2011, oil on linen, with a diagram of Antarctica's annual mean sea ice extent that shows it is increasing.

Welcome

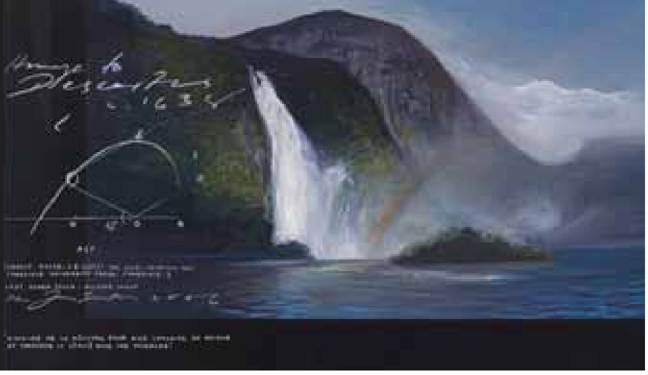
In this, our 11th issue, you'll find articles about people working on matroids, symmetry and C*-algebras. We also take a closer look at the CensusAtSchool project, interview a researcher who is examining what respect means in the mathematics classroom, and a professor of mathematics and art.

Marston Conder and Vaughan Jones
Co-Directors

INSIDE

New Zealand Institute of Mathematics & its Applications

taurangi

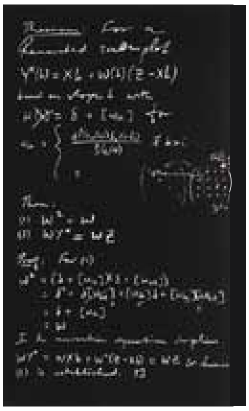


Homage to Descartes, 2006; Bowen Falls, Milford with overlays of the mathematics of rainbows.

◀ I in an address and get a value for a property based on sales and properties passed in at auction."

Like all teachers, Smith aimed to 'future-proof' his statistics teaching. "You teach students what boxplots look like, so they can recognise the analytical thinking that goes into that object when it changes to a new generation, and they can question it and invent a better one." However, he thinks there is a danger with "pressing a button on statistics software that works dynamically", because at the end they still may not know what the animation represents.

"Variability is very difficult for students to understand. I think it takes more than slick software."



Panel from Fading Light series, 2003, oil on linen, with research created on canvas before publication in the scientific literature.

Asking unpopular **questions**

Megan Clark is asking what respect means in the mathematics classroom, how senior secondary and undergraduate university teachers can collaborate, and other subversive questions.

Clark, who is head of the School of Mathematics, Statistics and Operations Research at Victoria University of Wellington, and her former PhD student Robin Averill are talking with secondary mathematics teachers and students from a range of schools and neighbourhoods about respectful behaviour in the classroom, and observing their interactions.

"There is clearly some miscommunication about it," says Clark. "Some behaviours that teachers might think as respectful, students may see as patronising, for example. Some discussion of a student's family life will be seen as intrusive and disrespectful by some students, who believe the teacher's role is only within their subject, but by other students as an indication that the teacher has bothered to find out about the whole person."

Clark is also working on better integration of senior secondary and early university maths education, with Professor Bill Barton at the University of Auckland, Professor Glenda Anthony from Massey University and Dr Alex James from Canterbury University. "Teams in Christchurch, Auckland, Palmerston North and Wellington will trial a programme designed by secondary and tertiary teachers in secondary schools that take part."

Clark says senior secondary mathematics and tertiary teachers have been more collaborative than in some other subjects; "University staff have been members of the NZ Association of Mathematics Teachers for years and regularly speak at conferences. Putting their perspectives together should make a richer experience for students."

In their exploration of the transition from high school to university mathematics courses as a rite of passage, she and Canadian colleague Miroslav Lovric have cast strong doubt on the worth of transition or bridging courses. "Students doing these courses do no better than students of the same background who didn't do them, which questions the prevailing assumptions. People assume because it's a nice thing to do, it's a good thing to do. At some stage students have to become independent learners, and I think we prop them up for far too long." This is unpopular talk, but not unusual from someone who likes to examine assumptions "that are never questioned, yet are the basis of practice".

Megan with her son Max in Hanoi.

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Census@school by and for students

Rachel Cunliffe, kneeling on left, with participating students from St Mary's School in Northcote; CensusAtSchool supporter Shane Cortese is at the back. Photo: Godfrey Boehnke, University of Auckland.

More Canterbury students took part in the biennial 2011 CensusAtSchool than in 2009, despite the earthquakes that led to the cancellation of the Government Census.

Teachers and students in years 5 to 13 at New Zealand schools still have until the end of the year to participate in the fifth of these national school surveys. Around 25,000 primary, intermediate and secondary students had completed it at the time of writing, answering a range of questions about themselves, which include body measurements, transport, school subjects, technology, favourite TV show, global issues and the Rugby World Cup.

Preliminary results show a slight increase in respondents whose favourite learning area is mathematics and statistics: from 7.76 percent in 2009 to 8.4 this year. The proportion of boys to girls whose favourite subject is maths has slightly increased since 2003 to almost double - 11.6 percent of boys compared to 6.1 percent of girls.

However, this difference is largely at primary school level. The parabolic curve in maths interest, with peaks at primary school and among 17-year-olds, has persisted in the 2011 respondents so far. Maths was a favourite of 19 percent of 8-year-olds, dropping to five percent of 14 and 15-year-olds, and rising to 10 percent of 17-year-olds.

CensusAtSchool fits into the statistics curriculum and teaches the value of statistics in everyday life through information, by and for the students. With some surveys, students vote on which questions they've suggested should be included, says Co-Director Rachel Cunliffe.

There are many constraints on questions; the demographic data is the same as the official Census; some questions need to be common across most years, and a small number must be asked so CensusAtSchool data can be compared internationally. "We also want timely questions," she says. "We try to have lots of different types of variables that are interesting to kids as well as being useful data, with nice relationships they can investigate."

"They love it because it's a different activity, and it's interactive. They're not just sitting at a computer - they're measuring each other, asking each other's opinions. They're really interested in the answers to the questions, and how they compare with other students in their class and overseas."

The measuring and collection of information for the survey can be done in a single lesson, and online responses can be in English and Maori. "Teachers get their class data back immediately, so they can start using it the next day if they want to." The survey is accompanied by 42 class activities.

"We provide the tools so that teachers and students can build their own tables and analyse data themselves," says Cunliffe. The online random sampler enables data to be viewed as animated graphs, while the data viewer page enables students to explore data relationships and comparisons.

"We do some data cleaning, but we're not rigorous," she says. "For example, in finger lengths there's a separate bell curve of students who entered their data in centimetres rather than millimetres, with tiny finger lengths, and the main bell curve, which has major spikes every five millimetres. Students were probably rounding it up to the nearest half centimetre, or they measured in centimetres, realised their mistake and just multiplied it up. That gives teachers a really good opportunity to discuss measurement problems and data cleaning."

CensusAtSchool is funded jointly by the Ministry of Education, Statistics NZ and the University of Auckland, which provides staff time for technical support by Department of Statistics Multimedia Manager Stephen Cope, for data cleaning and analysis. Data from 2011 will be released next term, and new data added until the end of this year.

See also

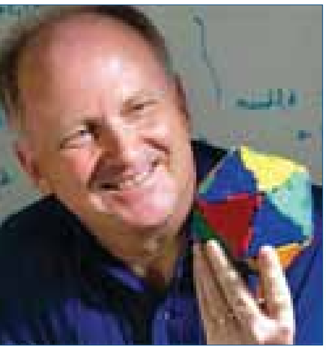
www.censusatschool.org.nz/

www.censusatschool.com/

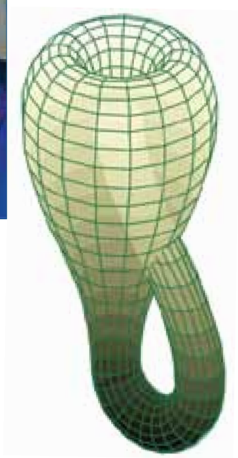
www.stats.govt.nz/tools_and_services/services/schools_corner.aspx

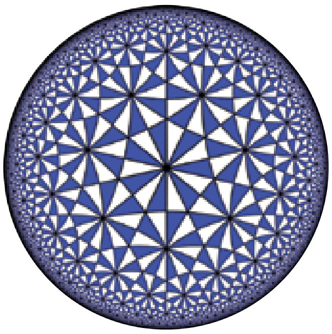
> **The mathematician's best work is art, a high perfect art, as daring as the most secret dreams of imagination, clear and limpid. Mathematical genius and artistic genius touch one another.**
>
> *Gosta Mittag-Leffler*

Computing symmetries networks and maps



When he is not co-directing the NZIMA, Professor Marston Conder uses combinatorial group theory and discrete computation to investigate symmetries of graphs, maps and surfaces.

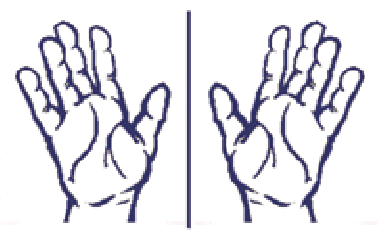




Above: A {3,7}-tessellation of the hyperbolic plane.

Top: A Klein bottle, a closed non-orientable surface.

Below: Hands are chiral - the mirror image cannot be superimposed on the original.



"One of my interests is the study of regular maps, which began with the platonic solids - regular convex polyhedra known to the ancient Greeks. If you expand each solid so that it becomes round, then its edges and vertices can be thought of as a highly symmetric network drawn on a sphere. Regular maps generalise platonic solids to surfaces of higher genus, like a torus or double torus.

"The more handles you stitch on, the higher the number of smooth holes that appear, which gives the genus - the genus of a sphere is 0. A PhD student and I used computers to determine all regular maps of genus 2 to 15, then five years ago with new computer methods I was able to extend it to genus 100, and in July I got up to genus 300."

Computers provide huge amounts of data, says Conder. "The data make it possible to see patterns that go on to infinity, and led me to the answers to some long-standing questions about regular maps." One was about chirality - when the map is different from its mirror images. On surfaces of genus 0 and 2, 3, 4, 5, and 6, there are no chiral regular maps. "I was able to prove this happens for infinitely many genera."

Other situations involving maximum symmetry that interest Conder include graphs, Riemann surfaces, hyperbolic manifolds, and closed non-orientable surfaces, in which inside and outside (or left and right) cannot be defined. The most well-known such surface is the Klein bottle.

"A question about structures on such surfaces is this: if you know it has a certain degree of symmetry, does it have more? Some colleagues in Spain and I solved this problem a few years ago for orientable surfaces - the sphere, torus, double torus and so on. Now I'm interested in doing the same thing for non-orientable surfaces."

Sometimes pure mathematicians make accidental discoveries, such as one which Conder happily stumbled upon five years ago. "You take a network, and prescribe in advance the maximum degree - the number of nodes any one node can be joined to, and the diameter - the largest number of steps needed to get from one node to another."

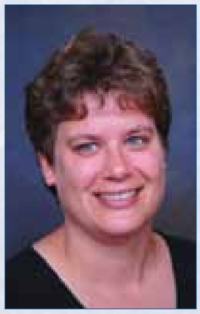
"For example, a cube has eight nodes, with three steps from one corner to its opposite, so diameter 3." Often the most efficient networks, which are interesting to computer scientists and engineers, have the largest amount of symmetry.

Conder used computers to find all the symmetric networks of degree 3 with up to 2,000 nodes. When he checked their diameter, he found a new one that is now the largest known network of degree 3 and diameter 10. He has since extended the census up to 10,000 nodes.

His most recent work is on polytopes, which are like regular maps in other dimensions. The smallest regular 2-D polytope is an equilateral triangle, with type {3}, and the smallest regular 3-D polytope is the tetrahedron, with type {3,3}: every node is on three edges, and every face has three edges. "A few people expected in that in higher dimensions the type would consist of all 3s. But the surprising thing is that from rank 9 onwards, the type of smallest example has all 4s."

"Some mathematicians don't like using computers to solve problems. But I find them really helpful in producing data, which often reveal patterns. Then I can try to prove that the patterns exist universally - beyond the data. This approach has shown me the way to a lot of general discoveries that have been surprising or unexpected, and I doubt would have been possible otherwise."

Examples from C*



Professor Astrid an Huef works in a relatively young branch of mathematics.

C*-algebra (pronounced C star) is a branch of mathematics concerning special kinds of operator algebras, which derived from attempts in the 1930s to understand the new and startling challenges of quantum mechanics. C*-algebra uses the tools of functional analysis to study problems associated with infinite-dimensional analogues of linear algebra, where the operators are linear transformations with extra properties.

Since the 1950s, operator algebras have provided a fundamental tool in areas such as representation theory, Fourier analysis and dynamics. More recently their use has led to surprising successes in other parts of mathematics, including algebraic topology, number theory, and combinatorics.

"Many of the operator algebras that I am interested in arise from dynamical systems - mathematical objects that model the way things change," says an Huef. "The challenge is to deduce information about the algebra from the dynamical system and vice versa. I'm interested in systems like transformation groups, groupoids and higher-rank graphs."

"There are very beautiful structure theories which relate properties of these dynamical systems to those of the associated algebras. Because we prove theorems that set up a one-to-one correspondence between properties of the system and properties of the algebra, I can tell colleagues what sort of dynamical system they need to start with to get the properties they want."

Most of an Huef's research has been about reversible dynamical systems, in which time can go backwards and forwards. More recently she has studied irreversible systems, which only go forwards. "We kept on finding interesting examples where the techniques developed for studying reversible systems were ineffective, and eventually decided to develop a general theory. We believe that this will become an increasingly important aspect of the subject. And it is easy to point to important irreversible processes: there was no one else working in this area of mathematics in New Zealand, and she was collaborating with colleagues from Australia, the United States, Scotland and Brazil.

She is now working with an algebraist in her new department "in a purely algebraic setting, which is new to me. New Zealand has some world-class algebraists, and I am looking forward to learning more about pure algebra." She has already built up a lively research group at Otago, and their working seminar is already producing interesting new results.

An Huef was born in Germany but "always had fond memories" of her two years as a teenager living in Wellington and going to Wellington High School. She has one female collaborator and says the number of women in pure mathematics is increasing. "I used to be the only one at some conferences, but now you need to queue for the women's bathroom."

$$||a^*a|| = ||a||^2$$

MATHEMATICAL EVENTS

21 - 24 November 2011, Silverstream, Hutt Valley
2011 NZ Mathematics and Statistics Postgraduate Conference http://msor.victoria.ac.nz/Events/NZMASP2011

27 Nov - 2 Dec, Rotorua
Volcanic DELTA: The 8th Southern Hemisphere Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics www.delta2011.co.nz/delta2011

6 - 8 December, University of Auckland
2011 New Zealand Mathematics Colloquium www.math.auckland.ac.nz/NZMC2011

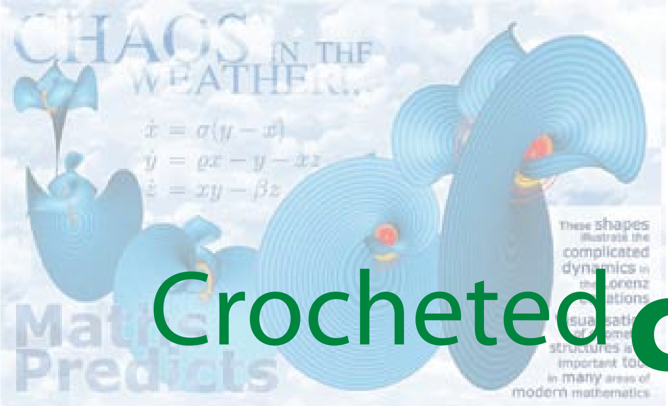
15 - 20 December, Victoria University of Wellington
The 12th Asian Logic Conference
http://msor.victoria.ac.nz/Events/ALC2011

8 - 13 January 2012, Tahunanui, Nelson
2012 NZMRI Summer Meeting, on Random Media and Random Walks
www.stat.auckland.ac.nz/~mholmes/workshop/

23 - 27 January, University of Auckland
New Zealand Probability Workshop 2012 www.stat.auckland.ac.nz/~mholmes/workshop/auckland_2012

29 January - 2 February, Warrnambool, Victoria, Australia
ANZIAM 2012 Conference http://anziam2012.monash.edu/

13 - 17 February, Queenstown
Conference and Magma workshop on Symmetries of Discrete Objects
www.math.auckland.ac.nz/~conder/SODO-2012/



Crocheted chaos

Dr Hinke Osinga, who will shortly be a Professor at the University of Auckland, has created a crochet model that illustrates chaos theory.

The piece (right) represents part of a surface in the classic Lorenz chaos system, constructed in 1963 by USA meteorologist Edward Lorenz in an attempt to understand why the weather was so hard to predict. Lorenz developed a simplified model of thermal convection in the atmosphere - as hot air rises, cools and falls it forms convection rolls, which can sometimes be seen as long lines of parallel cylindrical clouds.

Lorenz used three differential equations to model two neighbouring convection rolls. He was surprised to find that the equations behaved unpredictably: the smallest changes in starting co-ordinates led to very different evolution of the system after only a short time.

Despite their unpredictability, the equations produce a consistent structure. Whatever the starting co-ordinates, the system is attracted to spiralling sets of curves that form a butterfly shape with two wings. The motion on this chaotic Lorenz attractor is very unpredictable and continually swaps from one wing to another.

With her collaborator, Professor Bernd Krauskopf, Osinga tried to look at the system differently, to obtain "a more static, time-independent image of what is going on".

Using computer visualisation tools, they identified and animated a special surface of the system associated with the origin, where $x = y = z = 0$. Called the stable manifold of the origin, points on this surface do not go to the chaotic attractor, but converge to the origin instead. The system's chaotic dynamics are expressed through the complicated geometry of this special surface. "It is incredibly difficult to decide which points lie on it - and so don't exhibit chaos - and which points do not," says Osinga.

Krauskopf and Osinga spent years developing a computer algorithm that accurately constructed these surfaces. Then, suddenly, the pair realised that "the way that we computed the surface naturally translated into crochet instructions. When I saw that, I just had to try."

Osinga spent 85 hours crocheting more than 25,000 stitches, producing a metre-wide piece of the pancake. The form is held in shape by three wires, which represent the vertical z-axis; all the points with the same distance from the origin along the outer rim; and the only two solutions that meet perpendicular to the z-axis at the origin. "One major advantage of making a real crocheted model of the manifold is that it gives a better idea of the size of chaotic systems in real life than the tiny animations on the screen," she says.

"At the top you have a helical rotation going up, and horizontally there are two spiralling rotations going in opposite directions. Chaos is everywhere, which means that this 'pancake' folds over and fills space all around us. It is very, very long - you can think of it as a space-filling pancake, much like the examples we know of plane-filling curves."

The release of the crochet instructions in the *Mathematical Intelligencer* in 2004, which is read in many universities, colleges and high schools around the world, stimulated a global rash of crochet and sculpture about chaos. The crocheted Lorenz manifold has become an art object as well as a useful teaching tool.

See also

Mathematical Intelligencer crochet instructions: http://hdl.handle.net/1983/85

How the manifold spirals inside the attractor: ftp://ftp.aip.org/epaps/chaos/E-CHAOEH-9-024903/lorenz/inside.html

Above: Osinga and Krauskopf provided these pictures of the manifold for London Underground posters in World Mathematical Year 2000.

Awards and honours

John Butcher, one of our founding principal investigators, was presented with the 2011 Van Wijngaarden Award at a ceremony at the Centrum Wiskunde & Informatica in Amsterdam, in February.

Marston Conder, NZIMA Co-Director, has been appointed one of three non-European Associate Partners in a new EuroCoRE (Centre of Research Excellence), funded by the European Science Foundation, in the priority area of Graphs in Geometry and Algorithms. He has also been selected by the American and NZ Mathematical Societies as the first Maclaurin Lecturer, for a lecture tour of universities in the USA in 2012/13.

Adam Day, a former student of Rod Downey at Victoria University of Wellington who took part in the Algorithms programme and our summer meeting in January 2009, has won a prestigious Miller Fellowship to the University of California at Berkeley.

Rod Downey, one of our PIs, and **Andre Nies**, a key participant in our Logic and Computation programme, won the 2010 Shoenfield Prize of the Association for Symbolic Logic, jointly with Denis Hirschfeldt and Sebastiaan Terwijn, for their article *Calibrating Randomness*.

NZIMA Co-Director **Vaughan Jones** was appointed to a Distinguished Professorship at Vanderbilt University, Tennessee, from August 2011.

Shaun Hendy, Director of our programme on the Mathematics of the Nanosciences, has been elected incoming President of the New Zealand Association of Scientists. He also won the inaugural Massey University Distinguished Young Alumni Award, for alumni aged 35 and under.

$$x' = \sigma(y - x) \quad y' = \rho x - y - xz \quad z' = xy - \beta z$$

Photo: Steve Barker, Barker Photography

Variety in statistics

"Statistics is good for people with short attention spans," says Professor Thomas Lumley, deadpan. "In pure maths you might work on the same problem for 20 years. I admire that greatly but I would find it difficult. In statistics, what problems will be important tend to vary with the data."

His recent work illustrates this variety; it includes the R statistical language; air pollution; interactions between genes and environments; flow cytometry data about cancer-related particles; gene locations for blood-clotting proteins, kidney disease and heart function; HIV testing; and DNA resequencing.

Lumley's accent reflects his career moves - he grew up in Melbourne, studied in Oxford, and lived in Seattle for 12 years, before moving to Auckland in October to be Professor of Biostatistics at the University of Auckland. His link with Auckland started in his student days in the mid-90s, when he first sent bug fixes to Ross Ihaka and Rob Gentleman, developers of R. He has since written a book on analysis of complex surveys using R.

He says there's a demand for biostatisticians in New Zealand, "but that's true every-where". He enjoys solving the problems that come up in large medical studies. "In genetics studies where you're doing millions of statistical tests, you are working far out in the distribution curve - on the tails of the bell. If you're doing two million tests, you have to produce false positives at a very low rate, one in two million."

"You need to work with p values of .00000005 instead of .05, which is more usual. The difficulty is working out distribution theory for statistical tests with p values that small and several thousand people in the sample. And on the compu-tational side, if a test can go wrong one in a million times, when you're doing two million tests there is a 90 percent probability that it will go wrong!"

"Genetic technologies are getting cheaper and cheaper," he says, "so we will be able to use genetic data in more and more research. It's a very mathematical or computational area: DNA is a sequence of letters - closer to software than wetware."

Lumley says there is a lot of hype about genetics: "Treatment customised to your unique genetic makeup is not going to happen any time soon". Massive efforts like the Human Genome Project, which aims to identify the 25,000 or so genes in human DNA, and the HapMap project, which is cataloguing human genetic similarities and differences, have enabled researchers to find millions of genetic variants. Lumley was involved in one research project built on that information, which discovered a group of proteins that help regulate our heart rhythm. But the results from the big projects aren't useful for predicting individual disease, he says.

One statistical research area in which New Zealand punches above its weight is two-phase study design. "If you have a large sample of people, and you want to measure expensive additional information for a subset, New Zealand researchers are world leaders in finding the best way to design and analyse studies that get the most information for the money available."

Lumley is one of the contributors to StatsChat, a blog site commenting on stats in the news. Recent posts have discussed whether women take more sick leave, and the risks of anti-smoking drug Champix.

See also

www.statschat.org.nz

www.badscience.net, a UK blog about evidence-based medicine and statistics

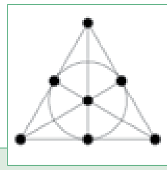
MATHS PROBLEM

DOES THERE EXIST A PROJECTIVE PLANE OF NON PRIME-POWER ORDER?

Simply: On an ordinary plane, two non-parallel lines intersect in a single point. The concept of a projective plane allows every two lines to intersect (for example, by adding a common point at infinity). The definition of a finite projective plane requires any two lines to intersect in exactly one point, but does not require the lines to be 'straight'.

In detail: A finite projective plane is a set of points, say $P = \{p_1,p_2,..,p_n\}$, and a set of lines, say $L = [l_1,l_2,...,l_n]$, where (a) every line contains the same number of points; (b) every point lies on the same number of lines; (c) every two points lie together on exactly one line; and (d) every two lines intersect in exactly one point.

Suppose each line contains k+1 points. An easy counting argument shows that each point lies on k+1 lines, and the properties (c) and (d) imply that the number of points and lines is $k^2 + k + 1$. In this case, we call the pair (P,L) a projective plane of order k.



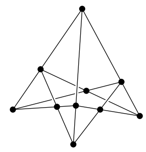
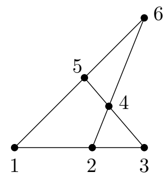
The Fano Plane, above, is a projective plane of order 2, the smallest such plane, with only seven points and seven lines.

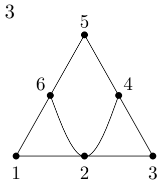
The problem is finding the possibilities for k. It is relatively easy to show that there exists a projective plane of order k whenever k is a prime number or a prime multiplied by itself a number of times, such as 4 (= 2^2) or 8 (= 2^3) or 1331 (= 11^3). It is not known if there are projective planes of order 12, 15, 18, 20, and so on.

Discipline: Linear algebra, projective geometry, combinatorics.

Progress: In 1901 Gaston Tarry proved that there is no projective plane of order 6, in answering Euler's 'Thirty-six officers problem' of 1782. The 1949 Bruck-Ryser theorem implies that there is no projective plane of order k if k–1 or k–2 is divisible by 4 and k is not the sum of two squares. In 1991, Clement Lam eliminated the possibility of a projective plane of order 10 with the help of a massive computer calculation.

NZIMA connection: This question was discussed at conferences in the NZIMA's programmes on Combinatorics and Geometry: Interactions with Algebra and Analysis.



Multiplying matroids

The study of matroids used to be slow-burning, but during the last decade many old problems have been solved and new techniques developed, according to Dr Dillon Mayhew at Victoria University of Wellington.

Matroids are unusual because there are 40 different ways to define these abstract objects, and no one definition is preferred. Some mathematicians think of them as graphs (networks of nodes joined by lines), others as matrices.

Mayhew is one of only 50 or so pure mathematicians worldwide studying this topic, and he prefers to think of matroids geometrically. "For me, a matroid is like a configuration of points sitting in space. It's projective geometry - if you project it onto a screen you get the same geometry, regardless of the angle of the screen. For example, points always sit on a common line or in the same plane. However, geometry is not always helpful; I can't visualise 10 points sitting in five-dimensional space."

Matroids were first explored in the 1930s, and underlie many different mathematical objects. A catalogue is useful for helping to find whether all matroids with property X also have property Y, or to find matroids with particular properties.

In 1969, with the help of an early computer, mathematicians catalogued the few thousand matroids with up to eight elements. Those researchers thought it "unlikely" that the complete set of matroids with nine elements would ever be compiled. However, it took only another 35 years before Mayhew and his Australian collaborator Gordon Royle created an online database of all 383,172.

They, too, were pessimistic about a catalogue of those with up to ten elements, saying that about 200 years of computing time would be needed to find the estimated two and a half trillion matroids. "I assumed it would be impossible for years, but another mathematician with clever optimisations has made good progress on the 10-element matroids, using a network of 30 or 40 computers for weeks on end," says Mayhew. However, he thinks "getting past 10 will be impossible for a long time".

Each number system has a different family of matroids, and obstacles mean that certain matroids will never arise in a particular number system. With five collaborators, Mayhew is also trying to work out how many obstacles exist in the five number system of matroids. This, too, has been a slow process.

In 1958, researchers proved that there is one obstacle in the two number system. It took 21 more years to prove the four obstacles in the three number system, and another 21 to prove the seven obstacles in the four number system. More elements means that the number of obstacles grows explosively: "We know that there are at least 564 obstacles, but we have no idea of the upper bound - it could be billions."

Matroids have applications in computer science, says Mayhew. "Binary matroids are points in space identified by strings of zeros and ones, so by studying binary matroids you understand space as a computer sees it. Understanding that space is really important for making error-correcting technology for CDs and satellites. Your satellite sends a string of ones and zeroes but interference swaps those digits; on the receiving end you need to work out where those errors occurred and correct them. The algorithms for doing that, and for correcting CD scratches, depend on understanding binary space."

Matroids are also useful for assessing the rigidity of joint and bar networks. "To decide if your joints are rigid or not, you have to look at a related matroid, and I think that's delightful. But the applications aren't the reason I study them – I think they're beautiful and fun."

$$r(X) + r(Y) \geq r(X \cup Y) + r(X \cap Y)$$