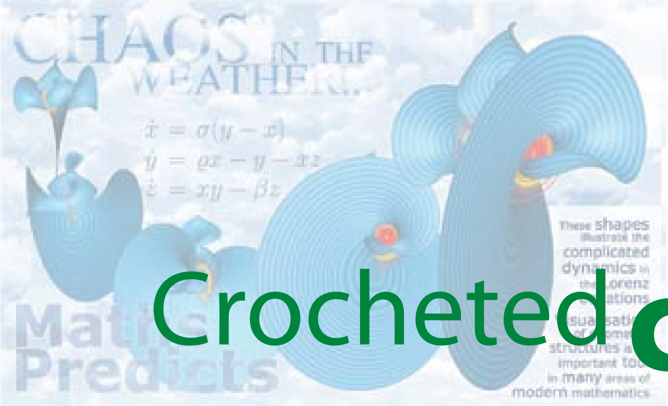


Crocheted chaos

Dr Hinke Osinga, who will shortly be a Professor at the University of Auckland, has created a crochet model that illustrates chaos theory.

The piece (right) represents part of a surface in the classic Lorenz chaos system, constructed in 1963 by USA meteorologist Edward Lorenz in an attempt to understand why the weather was so hard to predict. Lorenz developed a simplified model of thermal convection in the atmosphere - as hot air rises, cools and falls it forms convection rolls, which can sometimes be seen as long lines of parallel cylindrical clouds.

Lorenz used three differential equations to model two neighbouring convection rolls. He was surprised to find that the equations behaved unpredictably: the smallest changes in starting co-ordinates led to very different evolution of the system after only a short time.

Despite their unpredictability, the equations produce a consistent structure. Whatever the starting co-ordinates, the system is attracted to spiralling sets of curves that form a butterfly shape with two wings. The motion on this chaotic Lorenz attractor is very unpredictable and continually swaps from one wing to another.

With her collaborator, Professor Bernd Krauskopf, Osinga tried to look at the system differently, to obtain "a more static, time-independent image of what is going on".

Using computer visualisation tools, they identified and animated a special surface of the system associated with the origin, where $x = y = z = 0$. Called the stable manifold of the origin, points on this surface do not go to the chaotic attractor, but converge to the origin instead. The system's chaotic dynamics are expressed through the complicated geometry of this special surface. "It is incredibly difficult to decide which points lie on it - and so don't exhibit chaos - and which points do not," says Osinga.

Krauskopf and Osinga spent years developing a computer algorithm that accurately constructed these surfaces. Then, suddenly, the pair realised that "the way that we computed the surface naturally translated into crochet instructions. When I saw that, I just had to try."

Osinga spent 85 hours crocheting more than 25,000 stitches, producing a metre-wide piece of the pancake. The form is held in shape by three wires, which represent the vertical z-axis; all the points with the same distance from the origin along the outer rim; and the only two solutions that meet perpendicular to the z-axis at the origin. "One major advantage of making a real crocheted model of the manifold is that it gives a better idea of the size of chaotic systems in real life than the tiny animations on the screen," she says.

"At the top you have a helical rotation going up, and horizontally there are two spiralling rotations going in opposite directions. Chaos is everywhere, which means that this 'pancake' folds over and fills space all around us. It is very, very long - you can think of it as a space-filling pancake, much like the examples we know of plane-filling curves."

The release of the crochet instructions in the *Mathematical Intelligencer* in 2004, which is read in many universities, colleges and high schools around the world, stimulated a global rash of crochet and sculpture about chaos. The crocheted Lorenz manifold has become an art object as well as a useful teaching tool.

See also

Mathematical Intelligencer crochet instructions: http://hdl.handle.net/1983/85

How the manifold spirals inside the attractor: ftp://ftp.aip.org/epaps/chaos/E-CHAOEH-9-024903/lorenz/inside.html

Above: Osinga and Krauskopf provided these pictures of the manifold for London Underground posters in World Mathematical Year 2000.

Awards and honours

John Butcher, one of our founding principal investigators, was presented with the 2011 Van Wijngaarden Award at a ceremony at the Centrum Wiskunde & Informatica in Amsterdam, in February.

Marston Conder, NZIMA Co-Director, has been appointed one of three non-European Associate Partners in a new EuroCoRE (Centre of Research Excellence), funded by the European Science Foundation, in the priority area of Graphs in Geometry and Algorithms. He has also been selected by the American and NZ Mathematical Societies as the first Maclaurin Lecturer, for a lecture tour of universities in the USA in 2012/13.

Adam Day, a former student of Rod Downey at Victoria University of Wellington who took part in the Algorithms programme and our summer meeting in January 2009, has won a prestigious Miller Fellowship to the University of California at Berkeley.

Rod Downey, one of our PIs, and **Andre Nies**, a key participant in our Logic and Computation programme, won the 2010 Shoenfield Prize of the Association for Symbolic Logic, jointly with Denis Hirschfeldt and Sebastiaan Terwijn, for their article *Calibrating Randomness*.

NZIMA Co-Director **Vaughan Jones** was appointed to a Distinguished Professorship at Vanderbilt University, Tennessee, from August 2011.

Shaun Hendy, Director of our programme on the Mathematics of the Nanosciences, has been elected incoming President of the New Zealand Association of Scientists. He also won the inaugural Massey University Distinguished Young Alumni Award, for alumni aged 35 and under.

$x' = \sigma(y - x) \quad y' = \rho x - y - xz \quad z' = xy - \beta z$

Photo: Steve Barker, Barker Photography

Variety in statistics

"Statistics is good for people with short attention spans," says Professor Thomas Lumley, deadpan. "In pure maths you might work on the same problem for 20 years. I admire that greatly but I would find it difficult. In statistics, what problems will be important tend to vary with the data."

His recent work illustrates this variety; it includes the R statistical language; air pollution; interactions between genes and environments; flow cytometry data about cancer-related particles; gene locations for blood-clotting proteins, kidney disease and heart function; HIV testing; and DNA resequencing.

Lumley's accent reflects his career moves - he grew up in Melbourne, studied in Oxford, and lived in Seattle for 12 years, before moving to Auckland in October to be Professor of Biostatistics at the University of Auckland. His link with Auckland started in his student days in the mid-90s, when he first sent bug fixes to Ross Ihaka and Rob Gentleman, developers of R. He has since written a book on analysis of complex surveys using R.

He says there's a demand for biostatisticians in New Zealand, "but that's true everywhere". He enjoys solving the problems that come up in large medical studies. "In genetics studies where you're doing millions of statistical tests, you are working far out in the distribution curve - on the tails of the bell. If you're doing two million tests, you have to produce false positives at a very low rate, one in two million."

"You need to work with p values of .00000005 instead of .05, which is more usual. The difficulty is working out distribution theory for statistical tests with p values that small and several thousand people in the sample. And on the computational side, if a test can go wrong one in a million times, when you're doing two million tests there is a 90 percent probability that it will go wrong!"

"Genetic technologies are getting cheaper and cheaper," he says, "so we will be able to use genetic data in more and more research. It's a very mathematical or computational area: DNA is a sequence of letters - closer to software than wetware."

Lumley says there is a lot of hype about genetics: "Treatment customised to your unique genetic makeup is not going to happen any time soon". Massive efforts like the Human Genome Project, which aims to identify the 25,000 or so genes in human DNA, and the HapMap project, which is cataloguing human genetic similarities and differences, have enabled researchers to find millions of genetic variants. Lumley was involved in one research project built on that information, which discovered a group of proteins that help regulate our heart rhythm. But the results from the big projects aren't useful for predicting individual disease, he says.

One statistical research area in which New Zealand punches above its weight is two-phase study design. "If you have a large sample of people, and you want to measure expensive additional information for a subset, New Zealand researchers are world leaders in finding the best way to design and analyse studies that get the most information for the money available."

Lumley is one of the contributors to StatsChat, a blog site commenting on stats in the news. Recent posts have discussed whether women take more sick leave, and the risks of anti-smoking drug Champix.

See also

www.statschat.org.nz

www.badscience.net, a UK blog about evidence-based medicine and statistics

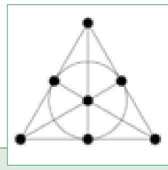
MATHS PROBLEM

DOES THERE EXIST A PROJECTIVE PLANE OF NON PRIME-POWER ORDER?

Simply: On an ordinary plane, two non-parallel lines intersect in a single point. The concept of a projective plane allows every two lines to intersect (for example, by adding a common point at infinity). The definition of a finite projective plane requires any two lines to intersect in exactly one point, but does not require the lines to be 'straight'.

In detail: A finite projective plane is a set of points, say $P = \{p_1,p_2,..,p_n\}$, and a set of lines, say $L = [l_1,l_2,...,l_n]$, where (a) every line contains the same number of points; (b) every point lies on the same number of lines; (c) every two points lie together on exactly one line; and (d) every two lines intersect in exactly one point.

Suppose each line contains k+1 points. An easy counting argument shows that each point lies on k+1 lines, and the properties (c) and (d) imply that the number of points and lines is $k^2 + k + 1$. In this case, we call the pair (P,L) a projective plane of order k.



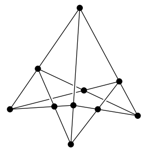
The Fano Plane, above, is a projective plane of order 2, the smallest such plane, with only seven points and seven lines.

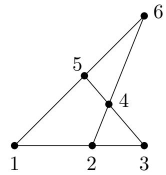
The problem is finding the possibilities for k. It is relatively easy to show that there exists a projective plane of order k whenever k is a prime number or a prime multiplied by itself a number of times, such as 4 ($= 2^2$) or 8 ($= 2^3$) or 1331 ($= 11^3$). It is not known if there are projective planes of order 12, 15, 18, 20, and so on.

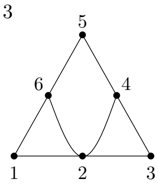
Discipline: Linear algebra, projective geometry, combinatorics.

Progress: In 1901 Gaston Tarry proved that there is no projective plane of order 6, in answering Euler's 'Thirty-six officers problem' of 1782. The 1949 Bruck-Ryser theorem implies that there is no projective plane of order k if k–1 or k–2 is divisible by 4 and k is not the sum of two squares. In 1991, Clement Lam eliminated the possibility of a projective plane of order 10 with the help of a massive computer calculation.

NZIMA connection: This question was discussed at conferences in the NZIMA's programmes on Combinatorics and Geometry: Interactions with Algebra and Analysis.







Multiplying matroids

The study of matroids used to be slow-burning, but during the last decade many old problems have been solved and new techniques developed, according to Dr Dillon Mayhew at Victoria University of Wellington.

Matroids are unusual because there are 40 different ways to define these abstract objects, and no one definition is preferred. Some mathematicians think of them as graphs (networks of nodes joined by lines), others as matrices.

Mayhew is one of only 50 or so pure mathematicians worldwide studying this topic, and he prefers to think of matroids geometrically. "For me, a matroid is like a configuration of points sitting in space. It's projective geometry - if you project it onto a screen you get the same geometry, regardless of the angle of the screen. For example, points always sit on a common line or in the same plane. However, geometry is not always helpful; I can't visualise 10 points sitting in five-dimensional space."

Matroids were first explored in the 1930s, and underlie many different mathematical objects. A catalogue is useful for helping to find whether all matroids with property X also have property Y, or to find matroids with particular properties.

In 1969, with the help of an early computer, mathematicians catalogued the few thousand matroids with up to eight elements. Those researchers thought it "unlikely" that the complete set of matroids with nine elements would ever be compiled. However, it took only another 35 years before Mayhew and his Australian collaborator Gordon Royle created an online database of all 383,172.

They, too, were pessimistic about a catalogue of those with up to ten elements, saying that about 200 years of computing time would be needed to find the estimated two and a half trillion matroids. "I assumed it would be impossible for years, but another mathematician with clever optimisations has made good progress on the 10-element matroids, using a network of 30 or 40 computers for weeks on end," says Mayhew. However, he thinks "getting past 10 will be impossible for a long time".

Each number system has a different family of matroids, and obstacles mean that certain matroids will never arise in a particular number system. With five collaborators, Mayhew is also trying to work out how many obstacles exist in the five number system of matroids. This, too, has been a slow process.

In 1958, researchers proved that there is one obstacle in the two number system. It took 21 more years to prove the four obstacles in the three number system, and another 21 to prove the seven obstacles in the four number system. More elements means that the number of obstacles grows explosively: "We know that there are at least 564 obstacles, but we have no idea of the upper bound - it could be billions."

Matroids have applications in computer science, says Mayhew. "Binary matroids are points in space identified by strings of zeros and ones, so by studying binary matroids you understand space as a computer sees it. Understanding that space is really important for making error-correcting technology for CDs and satellites. Your satellite sends a string of ones and zeroes but interference swaps those digits; on the receiving end you need to work out where those errors occurred and correct them. The algorithms for doing that, and for correcting CD scratches, depend on understanding binary space."

Matroids are also useful for assessing the rigidity of joint and bar networks. "To decide if your joints are rigid or not, you have to look at a related matroid, and I think that's delightful. But the applications aren't the reason I study them – I think they're beautiful and fun."

$$r(X) + r(Y) \geq r(X \cup Y) + r(X \cap Y)$$