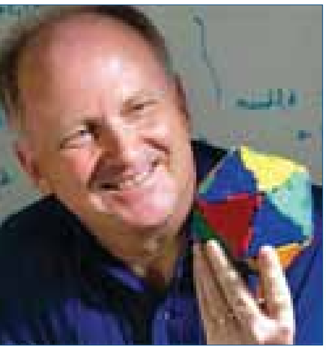
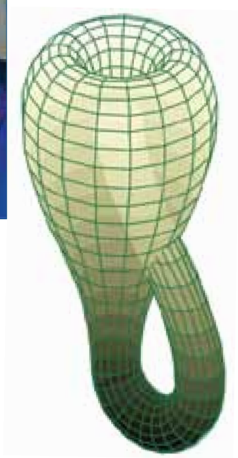
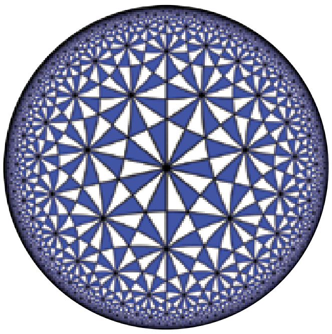
Computing symmetries networks and maps



When he is not co-directing the NZIMA, Professor Marston Conder uses combinatorial group theory and discrete computation to investigate symmetries of graphs, maps and surfaces.

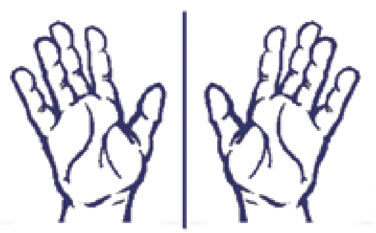




Above: A {3,7}-tessellation of the hyperbolic plane.

Top: A Klein bottle, a closed non-orientable surface.

Below: Hands are chiral - the mirror image cannot be superimposed on the original.



"One of my interests is the study of regular maps, which began with the platonic solids - regular convex polyhedra known to the ancient Greeks. If you expand each solid so that it becomes round, then its edges and vertices can be thought of as a highly symmetric network drawn on a sphere. Regular maps generalise platonic solids to surfaces of higher genus, like a torus or double torus.

"The more handles you stitch on, the higher the number of smooth holes that appear, which gives the genus - the genus of a sphere is 0. A PhD student and I used computers to determine all regular maps of genus 2 to 15, then five years ago with new computer methods I was able to extend it to genus 100, and in July I got up to genus 300."

Computers provide huge amounts of data, says Conder. "The data make it possible to see patterns that go on to infinity, and led me to the answers to some long-standing questions about regular maps." One was about chirality - when the map is different from its mirror images. On surfaces of genus 0 and 2, 3, 4, 5, and 6, there are no chiral regular maps. "I was able to prove this happens for infinitely many genera."

Other situations involving maximum symmetry that interest Conder include graphs, Riemann surfaces, hyperbolic manifolds, and closed non-orientable surfaces, in which inside and outside (or left and right) cannot be defined. The most well-known such surface is the Klein bottle.

"A question about structures on such surfaces is this: if you know it has a certain degree of symmetry, does it have more? Some colleagues in Spain and I solved this problem a few years ago for orientable surfaces - the sphere, torus, double torus and so on. Now I'm interested in doing the same thing for non-orientable surfaces."

Sometimes pure mathematicians make accidental discoveries, such as one which Conder happily stumbled upon five years ago. "You take a network, and prescribe in advance the maximum degree - the number of nodes any one node can be joined to, and the diameter - the largest number of steps needed to get from one node to another."

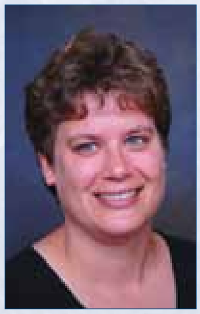
"For example, a cube has eight nodes, with three steps from one corner to its opposite, so diameter 3." Often the most efficient networks, which are interesting to computer scientists and engineers, have the largest amount of symmetry.

Conder used computers to find all the symmetric networks of degree 3 with up to 2,000 nodes. When he checked their diameter, he found a new one that is now the largest known network of degree 3 and diameter 10. He has since extended the census up to 10,000 nodes.

His most recent work is on polytopes, which are like regular maps in other dimensions. The smallest regular 2-D polytope is an equilateral triangle, with type {3}, and the smallest regular 3-D polytope is the tetrahedron, with type {3,3}: every node is on three edges, and every face has three edges. "A few people expected in that in higher dimensions the type would consist of all 3s. But the surprising thing is that from rank 9 onwards, the type of smallest example has all 4s."

"Some mathematicians don't like using computers to solve problems. But I find them really helpful in producing data, which often reveal patterns. Then I can try to prove that the patterns exist universally - beyond the data. This approach has shown me the way to a lot of general discoveries that have been surprising or unexpected, and I doubt would have been possible otherwise."

Examples from C*



Professor Astrid an Huef works in a relatively young branch of mathematics.

C*-algebra (pronounced C star) is a branch of mathematics concerning special kinds of operator algebras, which derived from attempts in the 1930s to understand the new and startling challenges of quantum mechanics. C*-algebra uses the tools of functional analysis to study problems associated with infinite-dimensional analogues of linear algebra, where the operators are linear transformations with extra properties.

Since the 1950s, operator algebras have provided a fundamental tool in areas such as representation theory, Fourier analysis and dynamics. More recently their use has led to surprising successes in other parts of mathematics, including algebraic topology, number theory, and combinatorics.

"Many of the operator algebras that I am interested in arise from dynamical systems - mathematical objects that model the way things change," says an Huef. "The challenge is to deduce information about the algebra from the dynamical system and vice versa. I'm interested in systems like transformation groups, groupoids and higher-rank graphs."

"There are very beautiful structure theories which relate properties of these dynamical systems to those of the associated algebras. Because we prove theorems that set up a one-to-one correspondence between properties of the system and properties of the algebra, I can tell colleagues what sort of dynamical system they need to start with to get the properties they want."

Most of an Huef's research has been about reversible dynamical systems, in which time can go backwards and forwards. More recently she has studied irreversible systems, which only go forwards. "We kept on finding interesting examples where the techniques developed for studying reversible systems were ineffective, and eventually decided to develop a general theory. We believe that this will become an increasingly important aspect of the subject. And it is easy to point to important irreversible processes: there was no one else working in this area of mathematics in New Zealand, and she was collaborating with colleagues from Australia, the United States, Scotland and Brazil.

She is now working with an algebraist in her new department "in a purely algebraic setting, which is new to me. New Zealand has some world-class algebraists, and I am looking forward to learning more about pure algebra." She has already built up a lively research group at Otago, and their working seminar is already producing interesting new results.

An Huef was born in Germany but "always had fond memories" of her two years as a teenager living in Wellington and going to Wellington High School. She has one female collaborator and says the number of women in pure mathematics is increasing. "I used to be the only one at some conferences, but now you need to queue for the women's bathroom."

MATHEMATICAL EVENTS

21 - 24 November 2011, Silverstream, Hutt Valley
2011 NZ Mathematics and Statistics Postgraduate Conference http://msor.victoria.ac.nz/Events/NZMASP2011

27 Nov - 2 Dec, Rotorua
Volcanic DELTA: The 8th Southern Hemisphere Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics www.delta2011.co.nz/delta2011

6 - 8 December, University of Auckland
2011 New Zealand Mathematics Colloquium www.math.auckland.ac.nz/NZMC2011

15 - 20 December, Victoria University of Wellington
The 12th Asian Logic Conference
http://msor.victoria.ac.nz/Events/ALC2011

8 - 13 January 2012, Tahunanui, Nelson
2012 NZMRI Summer Meeting, on Random Media and Random Walks
www.stat.auckland.ac.nz/~mholmes/workshop/

23 - 27 January, University of Auckland
New Zealand Probability Workshop 2012 www.stat.auckland.ac.nz/~mholmes/workshop/auckland_2012

29 January - 2 February, Warrnambool, Victoria, Australia
ANZIAM 2012 Conference http://anziam2012.monash.edu/

13 - 17 February, Queenstown
Conference and Magma workshop on Symmetries of Discrete Objects
www.math.auckland.ac.nz/~conder/SODO-2012/

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