

INSIDE



3 Managing weeds with maths



5 Milk tankers and traffic flows



6 Maths and Scottish country dancing



8 Quandles knots and manifolds



New Zealand Institute of Mathematics & its Applications

More than the boats are battling

America's Cup yachts are wrapped in mathematics. Jenny Rankine investigates.

Two powerful yachts strain, leaping half out of the water with each wave, sails the size of 747 wings taut with a massive press of air. For most spectators, it is a duel between two crews. Sponsors' brands are hoping for a win for their boat. For the backroom teams, the two boats are the products of particular suites of mathematical models, proven or defeated in the media glare of the America's Cup.

NZIMA programme leader Professor Andy Philpott, from the University of Auckland Engineering Science Department, has been involved in every America's Cup since he developed a velocity prediction programme for the 1995 Black Magic team. In 2007 in Spain, each boat will be guided by several different maths teams, some of them from the NZIMA.

The first maths team contributes to the yacht's design. They use computational fluid dynamics programs to compute the drag forces on particular hull designs. Similar codes are used to predict the lift and drag forces on sail designs. Velocity prediction programmes (VPP) take force models from sails and hulls and integrate those in a set of equations.

"So if you have a sail at this angle to the wind and this wind speed you get a certain set of forces," says Philpott. "If the boat is moving through the water at this speed, then another set of forces is acting on the hull. In each case the force vector and the moment vector both have three components. At an equilibrium speed, the sum of the forces and moments has to equal zero. Once these six equations are solved, the boat's speed can be predicted before it is built."

The second maths team concentrates on structural mechanics, trying to keep the weight as low as possible while ensuring the reliability of the boat under load. In 2007, this aspect is bound to get more attention from Emirates Team New Zealand after a broken mast fitting and hull breaches undermined their last cup defence.

Another maths team builds race modelling programs. They feed the information from the VPP into a program that pits two boats with different rigs and hulls racing against each other in a variety of simulated wind conditions to see how they perform. Philpott's student David Teirney did this for the 2000 Cup defence. Says Philpott: "If you run thousands of races with random winds, one boat will win, say, 55 percent of races. That tells you how much better one



Emirates Team New Zealand NZL84 in a practice race against Desafio Espanol ESP88 in July 2006. Photos: Chris Cameron, ETNZ.

Welcome

Welcome to this, the first full colour bulletin from the New Zealand Institute of Mathematics and its Applications (NZIMA).

The NZIMA is one of New Zealand's seven Centres of Research Excellence (CoREs), set up in 2002 with funding from the Tertiary Education Commission. It is hosted by the University of Auckland.

Modelled on similar mathematical research institutes in other countries (like the Fields Institute in Canada, MSRI, Berkeley, and the Newton Institute, UK) the NZIMA aims to foster research of the highest possible quality in the mathematical sciences in NZ, through the support of thematic programmes, research fellowships, scholarships, visitors, conferences and other stimulating activity.

We are also now embarking on a programme of greater outreach to the wider community. This bulletin will contain information about many of the NZIMA's activities and future opportunities. We hope you will find it interesting and informative.

Find out more from www.nzima.org

Marston Conder and Vaughan Jones
Co-Directors



Andy Philpott, top, and Hamish Shield.
Photos: **Godfrey Boehnke**



◀ I design feature will be than another."

A different group again looks at wind conditions on the course, to help the weather teams decide which side of the start line they should be on and which side of the course they should take on the first leg. Each team gets the same data from weather buoys on the Valencia course. "The weather teams for all the syndicates are populated by New Zealanders," says Philpott. Some teams are using meteorological models, which usually give a single point forecast; some produce wider ensemble forecasts. "So it's a battle of several models," says Philpott. Emirates Team New Zealand's current models remain under wraps, but the maths is no less hotly contested for being hidden behind the action.

Philpott's approach is to treat the wind conditions as a stochastic (random) process that evolves over time, and calibrate it to real weather observations. Using the Valencia data from Emirates Team New Zealand, NZIMA-supported Masters student Hamish Shield has implemented a stochastic wind field model, aiming to predict what the wind will be doing at any point on the course when the boat gets there.

The model uses a complex set of equations developed by Professor Shane Henderson at Cornell University, who worked with Philpott and Teirney on the 2000 defence. It combines with Teirney's match racing simulation program, and uses an Excel spreadsheet

and a macro to animate the results. Shield says the model enables a weather team to see the probability of one yacht reaching the first buoy first given a range of starting positions and wind conditions.

"The team with the best model will make the best decisions, get the better wind shifts and have a greater chance of winning," says Shield. His stochastic wind field model could be used for winds in any location, and for other applications such as wind farms. "It was definitely fun doing this; getting to work on this kind of project is a good reason to stay with maths."

See also

Philpott, A.B., Henderson, S.G., and Teirney, D.P., A simulation model for predicting yacht match-race outcomes, *Operations Research*, 52, 1, 1-16, 2004.

Philpott, A.B. Stochastic optimization in yacht racing, in *Applications of Stochastic Programming*, W. Ziemba and S. Wallace (ed.), SIAM, 2005.

Visiting lecture

Deborah Ball and Hyman Bass gave a series of lectures on mathematics and education during a recent visit to Auckland. Ball directs the Mathematics Teaching and Learning to Teach Project at the University of Michigan in Ann Arbor. Bass is a distinguished research mathematician with a strong interest in mathematics education. Photo: Jeremy Ralston



“Since you are now studying geometry and trigonometry, I will give you a problem. A ship sails the ocean. It left Boston with a cargo of wool. It grosses 200 tons. It is bound for Le Havre. The main mast is broken, the cabin boy is on deck, there are 12 passengers aboard, the wind is blowing east-north-east, the clock points to a quarter past three in the afternoon. It is the month of May. How old is the captain?” Gustave Flaubert, 1821-1880.

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Using **maths** to manage **weeds** and **invasive species**

Dr Jennifer Brown and her colleagues at the Biomathematics Research Centre at the University of Canterbury have recently started a three-year research programme, funded by the NZIMA, to investigate how current developments in mathematical and statistical research can help manage weeds and invasive species. Anna Meyer spoke to her.



From possums to dandelions, mice to didymo, New Zealand has more weeds and invasive species than almost any other country. Not only are they one of our most difficult conservation issues, they also cause widespread problems for agriculture and horticulture.

Fortunately, New Zealand is also a leader in conservation and weed management, with a variety of innovative techniques continually being developed to help keep these pests under control.

Says Dr Brown: "We have people doing a really good job at weed management, and we have mathematicians and statisticians who are developing mathematical and statistical tools which could be applied to this. But we needed to build that bridge between the two groups."

Consequently, a major part of the project is a five-day workshop, planned for April next year, which will bring together weed and conservation managers, and mathematicians and statisticians. "The idea is to put them in a room together for five days, and get them to talk to each other," says Dr Brown.

Weed and conservation managers will be encouraged to present the problems they are having, and mathematicians and statisticians will then be asked to suggest mathematical tools to help solve them.

This is the first time such an approach has been tried here. "Normally, its been the mathematicians and statisticians providing methods and then leaving it for the biologists to figure out how to use them. But we've twisted it round. We're saying, biologists, you ask the questions, and as mathematicians, we'll provide the goods," says Dr Brown.

Until now, there has been limited use of mathematics in weed management, because of the difficulty involved in adapting generic mathematical tools for the specialist needs of conservationists and

weed managers. "In this programme, we want to make mathematical models more accessible to biologists," she explains.

Some of the problems in weed management that mathematics could help with include: how can you predict where a weed is going to spread? How can new weed invasions be detected? How do we model weed growth and dispersal? How can limited budgets best be spent? And what will a particular weed population look like in 10 years time after a particular management strategy has taken place?

As well as the workshop, the project will involve research into several areas of weed management over the next three years, which will be carried out by a team of six to eight mathematicians and statisticians. A postdoctoral fellow has also recently joined the team, and two PhD students are starting next year.

Long term, Dr Brown would like to see a closer working relationship between mathematicians and weed managers. "I hope that in 10 years' time, every university will have one or two mathematicians working on weed management," she says.

"I want the mathematical community to realise this is a really exciting area of research to be working on. New Zealand is a world leader in weed management, so there's a lot of planning ahead at the moment, and innovative management. We're doing the best in the world, and we're just going to make that better."



Dr Brown, centre, with colleagues David Wall and Alex James. Photo: Eve Welch

Mathematics is often defined as the science of space and number ... it was not until the recent resonance of computers and mathematics that a more apt definition became fully evident: mathematics is the science of patterns.

Lynn Arthur Steen