NZ IMAGES

ISSUE ONE

More than the boats are battling

Emirates Team New Zealand NZL84 in a practice race against Desafio Espanol ESP88 in July 2006. Photos: Chris Cameron, ETNZ.

America's Cup yachts are wrapped in mathematics. Jenny Rankine investigates.

Two powerful yachts strain, leaping half out of the water with each wave, sails the size of 747 wings taut with a massive press of air. For most spectators, it is a duel between two crews. Sponsors' brands are hoping for a win for their boat. For the backroom teams, the two boats are the products of particular suites of mathematical models, proven or defeated in the media glare of the America's Cup.

NZIMA programme leader Professor Andy Philpott, from the University of Auckland Engineering Science Department, has been involved in every America's Cup since he developed a velocity prediction programme for the 1995 Black Magic team. In 2007 in Spain, each boat will be guided by several different maths teams, some of them from the NZIMA.

The first maths team contributes to the yacht's design. They use computational fluid dynamics programs to compute the drag forces on particular hull designs. Similar codes are used to predict the lift and drag forces on sail designs. Velocity prediction programmes (VPP) take force models from sails and hulls and integrate those in a set of equations.

"So if you have a sail at this angle to the wind and this wind speed you get a certain set of forces," says Philpott. "If the boat is moving through the water at this speed, then another set of forces is acting on the hull. In each case the force vector and the moment vector both have three components. At an equilibrium speed, the sum of the forces and moments has to equal zero. Once these six equations are solved, the boat's speed can be predicted before it is built."

The second maths team concentrates on structural mechanics, trying to keep the weight as low as possible while ensuring the reliability of the boat under load. In 2007, this aspect is bound to get more attention from Emirates Team New Zealand after a broken mast fitting and hull breaches undermined their last cup defence.

Another maths team builds race modelling programs. They feed the information from the VPP into a program that pits two boats with different rigs and hulls racing against each other in a variety of simulated wind conditions to see how they perform. Philpott's student David Teirney did this for the 2000 Cup defence. Says Philpott: "If you run thousands of races with random winds, one boat will win, say, 55 percent of races. That tells you how much better one

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Welcome

Welcome to this, the first full colour bulletin from the New Zealand Institute of Mathematics and its Applications (NZIMA).

The NZIMA is one of New Zealand's seven Centres of Research Excellence (CoREs), set up in 2002 with funding from the Tertiary Education Commission. It is hosted by the University of Auckland.

Modelled on similar mathematical research institutes in other countries (like the Fields Institute in Canada, MSRI, Berkeley, and the Newton Institute, UK) the NZIMA aims to foster research of the highest possible quality in the mathematical sciences in NZ, through the support of thematic programmes, research fellowships, scholarships, visitors, conferences and other stimulating activity.

We are also now embarking on a programme of greater outreach to the wider community. This bulletin will contain information about many of the NZIMA's activities and future opportunities. We hope you will find it interesting and informative.

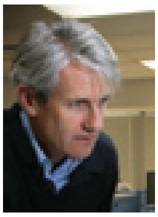
Find out more from www.nzima.org

Marston Conder and Vaughan Jones
Co-Directors

INSIDE

New Zealand Institute of Mathematics & its Applications

taurangi



Andy Philpott, top, and Hamish Shield. Photos: Godfrey Boehnke

◀ I design feature will be than another."

A different group again looks at wind conditions on the course, to help the weather teams decide which side of the start line they should be on and which side of the course they should take on the first leg. Each team gets the same data from weather buoys on the Valencia course. "The weather teams for all the syndicates are populated by New Zealanders," says Philpott. Some teams are using meteorological models, which usually give a single point forecast; some produce wider ensemble forecasts. "So it's a battle of several models," says Philpott. Emirates Team New Zealand's current models remain under wraps, but the maths is no less hotly contested for being hidden behind the action.

Philpott's approach is to treat the wind conditions as a stochastic (random) process that evolves over time, and calibrate it to real weather observations. Using the Valencia data from Emirates Team New Zealand, NZIMA-supported Masters student Hamish Shield has implemented a stochastic wind field model, aiming to predict what the wind will be doing at any point on the course when the boat gets there.

The model uses a complex set of equations developed by Professor Shane Henderson at Cornell University, who worked with Philpott and Teirney on the 2000 defence. It combines with Teirney's match racing simulation program, and uses an Excel spreadsheet and a macro to animate the results. Shield says the model enables a weather team to see the probability of one yacht reaching the first buoy first given a range of starting positions and wind conditions.

"The team with the best model will make the best decisions, get the better wind shifts and have a greater chance of winning," says Shield. His stochastic wind field model could be used for winds in any location, and for other applications such as wind farms. "It was definitely fun doing this; getting to work on this kind of project is a good reason to stay with maths."

See also

Philpott, A.B., Henderson, S.G., and Teirney, D.P., A simulation model for predicting yacht match-race outcomes, Operations Research, 52, 1, 1-16, 2004.

Philpott, A.B. Stochastic optimization in yacht racing, in Applications of Stochastic Programming, W. Ziemba and S. Wallace (ed.), SIAM, 2005.

Visiting lecture

Deborah Ball and Hyman Bass gave a series of lectures on mathematics and education during a recent visit to Auckland. Ball directs the Mathematics Teaching and Learning to Teach Project at the University of Michigan in Ann Arbor. Bass is a distinguished research mathematician with a strong interest in mathematics education. Photo: Geremy Ralston

"Since you are now studying geometry and trigonometry, I will give you a problem. A ship sails the ocean. It left Boston with a cargo of wool. It grosses 200 tons. It is bound for Le Havre. The main mast is broken, the cabin boy is on deck, there are 12 passengers aboard, the wind is blowing east-north-east, the clock points to a quarter past three in the afternoon. It is the month of May. How old is the captain?" Gustave Flaubert, 1821-1880.

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Using maths to manage weeds and invasive species

Dr Jennifer Brown and her colleagues at the Biomathematics Research Centre at the University of Canterbury have recently started a three-year research programme, funded by the NZIMA, to investigate how current developments in mathematical and statistical research can help manage weeds and invasive species. Anna Meyer spoke to her.

From possums to dandelions, mice to didymo, New Zealand has more weeds and invasive species than almost any other country. Not only are they one of our most difficult conservation issues, they also cause widespread problems for agriculture and horticulture. Fortunately, New Zealand is also a leader in conservation and weed management, with a variety of innovative techniques continually being developed to help keep these pests under control.

Says Dr Brown: "We have people doing a really good job at weed management, and we have mathematicians and statisticians who are developing mathematical and statistical tools which could be applied to this. But we needed to build that bridge between the two groups."

Consequently, a major part of the project is a five-day workshop, planned for April next year, which will bring together weed and conservation managers, and mathematicians and statisticians. "The idea is to put them in a room together for five days, and get them to talk to each other," says Dr Brown.

Weed and conservation managers will be encouraged to present the problems they are having, and mathematicians and statisticians will then be asked to suggest mathematical tools to help solve them.

This is the first time such an approach has been tried here. "Normally, its been the mathematicians and statisticians providing methods and then leaving it for the biologists to figure out how to use them. But we've twisted it round. We're saying, biologists, you ask the questions, and as mathematicians, we'll provide the goods," says Dr Brown.

Until now, there has been limited use of mathematics in weed management, because of the difficulty involved in adapting generic mathematical tools for the specialist needs of conservationists and weed managers. "In this programme, we want to make mathematical models more accessible to biologists," she explains.

Some of the problems in weed management that mathematics could help with include: how can you predict where a weed is going to spread? How can new weed invasions be detected? How do we model weed growth and dispersal? How can limited budgets best be spent? And what will a particular weed population look like in 10 years time after a particular management strategy has taken place?

Dr Brown, centre, with colleagues David Wall and Alex James. Photo: Eve Welch

As well as the workshop, the project will involve research into several areas of weed management over the next three years, which will be carried out by a team of six to eight mathematicians and statisticians. A postdoctoral fellow has also recently joined the team, and two PhD students are starting next year.

Long term, Dr Brown would like to see a closer working relationship between mathematicians and weed managers. "I hope that in 10 years' time, every university will have one or two mathematicians working on weed management," she says.

"I want the mathematical community to realise this is a really exciting area of research to be working on. New Zealand is a world leader in weed management, so there's a lot of planning ahead at the moment, and innovative management. We're doing the best in the world, and we're just going to make that better."

> **"Mathematics is often defined as the science of space and number ... it was not until the recent resonance of computers and mathematics that a more apt definition became fully evident: mathematics is the science of patterns."**
>
> *Lynn Arthur Steen*

Marvellous **motion:** predicting the behaviour of large systems

Accurately predicting how large systems behave – how they move and change over time – is very desirable in many areas of physics and chemistry. This has historically been very difficult, largely due to the need for accurate mathematical methods. Anna Meyer investigates.

Robert McLachlan
Photo: Graeme Brown

Large, complicated systems are everywhere around us – from the motion of planets in the solar system, to the collision of molecules as they undergo a chemical reaction, to the ever-changing patterns of weather and climate. Over the last ten years, Robert McLachlan, a professor of applied mathematics at Massey University and a Maclaurin Fellow of the NZIMA in 2005 has been developing inventive new mathematical tools to make it easier to study the behaviour of large systems.

To track how large systems behave, a branch of maths known as numerical integration is used, which involves calculating and predicting movement in the system over time as a series of tiny steps, building up to an overall picture. The problem is that although the starting positions and forces can be determined accurately using the laws of physics, small errors are unavoidably introduced at each step as the movement trajectory is calculated. These errors, due to intrinsic properties of the equations used, build up over time, and so even a tiny uncertainty early on becomes magnified very rapidly. This means that it has traditionally not been possible to accurately predict the behaviour of large systems very far into the future. This was particularly true for the motion of atoms or planets, where calculation errors are usually very large.

A solution to the problem was hit upon almost by accident a little over a decade ago. "It turns out that applied scientists had been ignoring what the numerical analysts were saying, and doing enormously long runs millions of years into the future, without worrying about the errors much," says Professor McLachlan. What is more, they were getting excellent results. Intrigued, mathematicians began to study the methods scientists had devised, and discovered they had hit upon a technique that at last gave a reliable way to study motion in large systems.

Nicknamed 'leapfrog', the method is the simplest example of what has now grown into an entire class of mathematics for studying large systems, called geometric integration. The method allows the overall behaviour of large systems be studied over long periods of time. Although errors are still introduced along the way, they do not affect the overall pattern that emerges for how the system behaves. This is because the methods preserve some aspects of the physical laws exactly, with no error, for example, the conservation of energy, momentum or symmetry, or the so-called 'symplectic' property, which couples position and velocity.

Professor McLachlan's research involves designing new geometric integration methods, and studying their behaviour. The idea is to develop techniques that are faster, more reliable and simpler, and that also have a variety of specific, desirable properties.

Many geometric integration methods are now known, and are being applied to problems such as predicting whether planets will remain in their current orbits, and how proteins fold into their final shape from their amino acid sequences. In the future it may even be possible to solve more difficult problems, such as modelling climate change – a breakthrough, no doubt, that would be well received.

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PRIMA donor

The NZIMA has become a founding member of the Pacific Rim Mathematical Association (PRIMA). This association of mathematical sciences institutes, departments and societies from around the Pacific Rim was established in 2005 to promote the development of the mathematical sciences throughout the region.

PRIMA aims to encourage wider participation in scientific activities in the Pacific Rim, share expertise and resources in the promotion of the mathematical sciences and their impact on society and the global economy, and to create a network for the exchange of ideas and the dissemination of scientific knowledge.

One of the first PRIMA initiatives is the encouragement of participation in summer schools and other meetings at reduced cost by students of its member institutions. Students from New Zealand universities affiliated with the NZIMA are eligible for subsidies.

See the PRIMA website at www.primath.org.

Milk tankers and traffic flows

Jenny Rankine explores some of the topics discussed at the NZIMA Workshop on Mathematical Models for Optimising Transportation Services in April 2005.

Milk tankers are such a regular feature in the life of dairying regions that few people give them a second thought. Farmer Smith expects one every morning, but every morning her herd produces a slightly different amount of milk. By the time the tanker gets to the Wihongi farm at the end of the road, it may not have enough space for the output of their cows, and the scheduler may have to send another tanker.

Arranging the collection of an unknown amount of milk from 12,000 farms and delivering it to more than 30 factories, without many such backtracks, is a constant headache for Fonterra's schedulers. They face it twice a day, every day of the year. Fonterra's milk collection consultant, Simon Harrison, presented the problem to the workshop and was heartened by the results.

He introduced the company's internal debate between stochastic and deterministic optimisation approaches. "A deterministic solution will always be at the boundary of existing constraints," he says, "and therefore has the biggest risk of being wrong on the day. The question is whether you can design a stochastic optimisation approach to reduce the risk of being wrong on the day at minimal cost to the mathematically optimal solution. For a business like Fonterra, that is the crux of the argument." At the time of the conference, Fonterra was evaluating software using the different optimisation models.

The workshop brought together a group of local and international scheduling experts to debate the issues in a Fonterra-sponsored panel session. "It was quite fruitful," says Harrison. "What we got from the workshop was a much better understanding of the issues, a set of questions to ask our prospective suppliers, and better tools to make sure that the path we're on is going to give us the results we're looking for." He says Fonterra would be interested in taking part in a similar session on a high priority maths problem again.

An unexpected result was that Fonterra's schedulers heard "all those brainy people struggling to model 150 pickups and went away thinking they do a really good job".

The workshop also included a stream about regional transport models, with a presentation by the Auckland Regional Transport Authority. Another expert panel considered transport models as tools for evaluating regional transport policies; how to increase Auckland public transport's share of travel time at reasonable cost; and how to plan for travel growth when forecast data is uncertain.

Workshop co-organiser Professor Andy Philpott says that traffic system models look at many individual drivers going from their origin to their destination and aiming for the quickest trip. "The models seek to construct an equilibrium where each driver is travelling by the route that gets them where they are going in the shortest time accounting for the similarly optimal choices of all the other drivers." The models allow researchers to explore the likely effect on traffic flows of changes such as expanding the capacity of a main road or motorway.

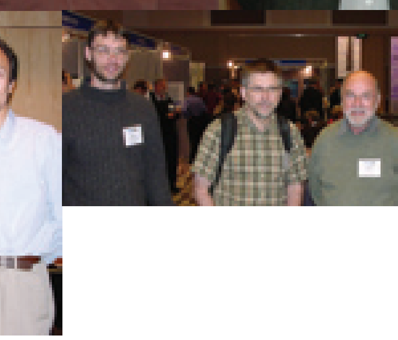
At the time, the introduction of tolls for Auckland central traffic was being debated. Professor Mike Florian from the University of Montreal told television viewers that tolls work only if there are alternative untolled routes that could then get more congested. They may not have any effect on drivers when there are no alternative routes.

A third theme of the four-day workshop was transport pricing and revenue management. The event was attended by more than 80 people from Australia, Canada, Chile, China, Denmark, Germany, India, Israel, the Netherlands, Norway, Singapore, Spain, Sweden, USA and New Zealand. It was hosted by the Department of Engineering Science at the University of Auckland and funded by NZIMA and the University's Operations Research Group. See www.esc.auckland.ac.nz/Transportation.

ASC/NZSA 2006

This joint conference of the Statistical Society of Australia Inc (SSAI) and the New Zealand Statistical Association (NZSA) was held in Auckland from July 3 to 6. Top-class keynote speakers led a rich programme of 48 invited talks, 150 contributed talks and 16 poster sessions for the 290 participants. See more programme details at www.statsnz2006.com.

Photos: Harold Henderson and Rod Ball



The dance of mathematics

At first glance, a connection between Scottish Country Dancing and complex mathematics is not immediately obvious. Anna Meyer investigates.

Developed in the 18th century, Scottish Country Dancing is believed to be derived from English country dancing, brought to Scotland by the gentry who had enjoyed it while on holiday. In this social form of dance that predates modern ballroom dancing, groups of couples follow precise, progressive footwork patterns, accompanied by different types of music.

In a classic example of the subtle relationship between maths and art, Rod Downey, a Professor of Mathematics at Victoria University and the first Maclaurin Fellow of the NZIMA, uses his mathematical work as inspiration for the dances he writes and performs as part of his favourite hobby.

Professor Downey's research involves understanding algorithmic processes, a discipline that has applications in many areas, particularly computer science. "An algorithm is a recipe for doing something," he explains. "There are a lot of theorems you can prove, but if you want to implement them in some form, say on a computer, you need an algorithm, so the computer can execute them in little steps. For example, when you turn your computer on, sitting behind there are algorithms. I guess you could call it on the borderline between mathematics and computer science."

Look closer, and it becomes clear that Scottish country dances bear a striking resemblance to algorithms – indeed, to mathematics as a whole. Dances are written as a series of logical steps that participants must follow sequentially, and numbers are everywhere – in the number of couples, the precise formations they dance, and how the dance steps relate to the timing of the music.

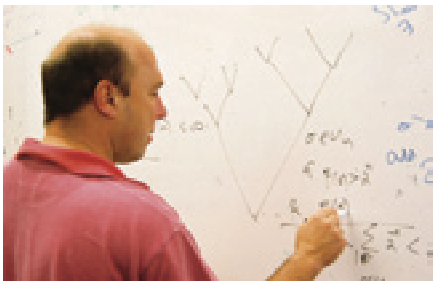
"When you devise dances, you have to think about things moving in space, visualise what's going on," Professor Downey explains. "With a lot of the mathematics I do, you have to do the same kind of thing. I do phrasing and patterns in dances rather similarly to doing proofs – it's just that it's a little bit easier."

Mathematicians, he believes, actually make some of the best choreographers. "It's noticeable that a lot of the best dance devisors down through the years have actually been mathematicians. For example, Hugh Foss was one of the original devisors of modern dances, and he was a well-known mathematician who worked at Bletchley Park, decoding."

Professor Downey began dancing on the suggestion of his wife, Kristin, who had learned the hobby while living in Singapore. "I played a lot of sport when I was young, and I kept on getting injured, so my wife said "why don't you come along and see what it's like?" I went along and I liked it."

Now a qualified teacher, he has written a book of new dances, The Cane Toad Collection, and is working on a second one. Dance titles that include They Stole My Wife From Me Last Night, Jill's Dental Jig, and Buttermilk Falls, reflect the fact that the dances are full of personality and interest.

When not at work or involved in dancing, Professor Downey can often be found indulging in his other favourite hobby – surfing at Makara Point or in the Wairarapa. This, however, does not have a maths basis. "That's purely just for pleasure," he says.



Robert Downey
Photo: Myles Herschell, drawn from Revisioning Science, a photo-essay project developed by Massey University and funded by the Government's Science and Technology Promotion Fund.

"Black holes are where God divided by zero."
Steven Wright

MATHEMATICAL EVENTS

4-6 December 2006, Hamilton
NZ Mathematics Colloquium
Contact: Stephen Joe, Mathematics Department, University of Waikato, stephenj@math.waikato.ac.nz

8-13 January 2007, Bay of Islands
NZIMA Summer Workshop on partial differential equations: Analysis, applications and inverse problems www.math.auckland.ac.nz/~fox/SummerWorkshop.html

28 January to 1 February, Fremantle, WA
ANZIAM 2007, annual conference of the professional association for industrial and applied mathematics in Australia and New Zealand
www.anziam07.murdoch.edu.au/

29 January to 2 February, Fiji
SPM07 Second South Pacific Conference on Mathematics
www.riemann.usp.ac.fj/~spcm07/

5-9 February, Wollongong, NSW, Australia
MISG-07 Mathematics-in-Industry Study Group 2007
* Cost reduction for NZ students affiliated with the NZIMA*
www.misg.math.uow.edu.au/

16-20 April, Hanmer Springs
NZIMA Programme Workshop on Modelling Invasive Species and Weed Impact
www.math.canterbury.ac.nz/bio/NZIMA/

MATHS AT TAIPA

Quandles
knots & manifolds

Topology is a pure maths discipline, but a focus of the NZIMA Summer Meeting on topology in January was the cracking of current banking and financial encryptions. Jenny Rankine investigates.

Meeting speakers Michael Freedman and Kevin Walker, of Microsoft, explored topological quantum field theory, developed originally to measure the set of all loops on a surface – what topologists call the fundamental group. They discussed the potential of these coding methods for breaking cryptographic systems based on integer factorisation, widely used in banking.

Co-director of the NZIMA programme on Geometric Methods in the Topology of 3-Dimensional Manifolds, Professor David Gauld, of the University of Auckland, said the Taipa meeting this year attracted 60 people, from Australia, Japan, North America, the UK and around New Zealand.

The meetings were initiated by programme co-director Professor Vaughan Jones in 1994, and focus on a different branch of mathematics each year. Until this year Professor Jones was the southern hemisphere's only Fields Medallist, the maths equivalent of the Nobel Prize. It was awarded to him in 1990 by the International Mathematical Union for a new polynomial knot invariant (an object that distinguishes one theoretical knot from another), which was named after him.

Topology is sometimes called rubber sheet geometry, because it concerns itself with the spatial properties that are preserved after shapes are stretched or deformed without breaking. It does not distinguish between a square and a circle (as a rubber band circle can be stretched into a square) and it ignores distances (so that two different sized circles are equivalent in a topological universe).

Professor Gauld is studying manifolds – abstract mathematical spaces – that are too big to measure; some physicists think the universe is an example. "If we're trying to apply the maths to the universe, there's a long way to go," he says. "I'm still dabbling in two dimensions and physicists want solutions for three to ten dimensions."

The programme's third co-director, Dr Roger Fenn, of the University of Sussex, is working with PhD student Stevie Budden on quandles. These algebraic objects give rise to knot invariants. BSc honours student Michael Brough is doing his dissertation on a new knot invariant. Applications of knot theory include DNA recombination.

"People say pure mathematicians are just playing games with bunches of rules," says Gauld. "The amazing thing is that so often, ten or 50 years later, these great applications arise. When I first heard about topological quantum field theory in 1994, there was no mention of their connection with banking encryptions."

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NOTABLE MATHS PROBLEMS

POINCARÉ CONJECTURE

(1900) If M is a 3-manifold with trivial fundamental group, and Π_i(M)=0 for i=1,2 and =Z for i=0,3 (ie, M has the homotopy groups of a 3-sphere), then M is homeomorphic to the 3-sphere.

Simply: (1904) That if any loop on the surface of a three-dimensional shape can be shrunk to a point (as any loop can on a 3-D sphere) then the shape is just a 3-D sphere.

Discipline: Topology

Originator: Jules Henri Poincaré, 1854-1912.

Incentive: $US1million, one of the seven Millennium Prize Problems of the Clay Mathematics Institute.

Notable false proof: JHC Whitehead, 1934.

Has led to: Interesting new examples of 3-manifolds; several celebrated cases of Poincaritis.

Unusual aspect: Solving this problem in four and more dimensions has been much easier than solving it in three.

Likely proof: Grigori Perelman, Steklov Institute of Mathematics, St Petersberg, 2002 and 2003, although the Clay prize has yet to be awarded.

NZIMA programme connection: Geometric Methods in the Topology of 3-Dimensional Manifolds.

Meeting participants Gabriela Slezakova and Qing Zhang, students at Waikato and Massey. Right: Vaughan Jones.