

INSIDE



3 Ice, energy and wood



4 Sampling southern weed pests



7 Harnessing waves and tides



8 Analysing earthquake signals



New Zealand Institute of Mathematics & its Applications



Aircraft designer and hopeful astronaut

Auckland-born Karen Willcox is taking a sabbatical in her home town from her job as a professor of aeronautics and astronautics at the Massachusetts Institute of Technology on the USA's east coast. She spoke with Jenny Rankine.

Willcox's optimisation work has been one of the forces behind two revolutionary aircraft designs - the unmanned X-48B blended wing body (BWB), and the Silent Aircraft Initiative (SAI) conceptual design that may be flying commercially by 2030.

The Boeing BWB's integration of wings, fuselage, engines and tail makes it 30 percent more fuel efficient than a similar-sized conventional aircraft. The 1/12 scale prototype has a 21-foot wingspan, and is scheduled for remotely-piloted flight tests at the end of the year. The US Air Force says it could be in service within 15 years as a multi-role, long-range military aircraft. The 20-person design group for the Boeing BWB included specialists in aerodynamics, structures, engine, propulsion, weights and control systems as well as optimisation.

The SAI design is also triangular, with engines mounted above the wings to shield noise from the ground. Willcox led the SAI's design and acoustic integration research component.

She also leads an international research effort to cut fuel consumption and carbon emissions in 747-size aircraft, with Boeing, NASA, Purdue and Stanford Universities and MIT. While aircraft contribute about four percent of global human carbon emissions, "it is still important for aircraft to reduce that," she says. "Commercial aircraft operate for up to 30 years, so any reductions we make now will have an impact."

"We are looking at the whole design process and systems integration from aerodynamics to better controls and smart computers to change the way an aircraft uses fuel dramatically. There isn't a single technology to achieve the results we want."

"The kinds of maths students learn in high school is everywhere in this project. At the heart of optimisation are derivatives, which tell us how much change we will see in the aircraft's weight or load if we change some design parameter slightly."

These projects use multi-disciplinary design optimisation (MDO). "We write the design as an optimisation problem: minimise the weight of the aircraft subject to these constraints - for example, that the wings don't break, drag equals thrust, lift equals weight, noise doesn't exceed certain limits on take-off and landing - thousands of constraints."

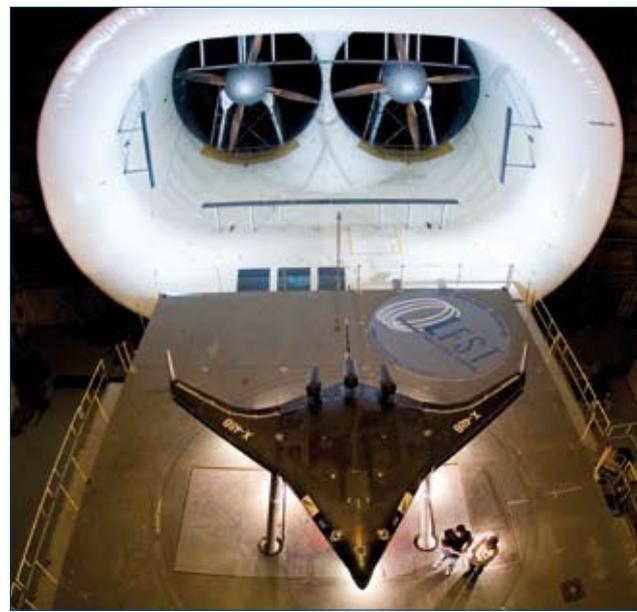
"We'd set the computer running at night and come back in the morning and it would have a design. We'd take it back to the specialists and there might be something not quite right; we were constantly re- ▶

Top: Willcox in the 'vomit comet', Nasa's zero-gravity aircraft. Below: The BWB prototype. Photo: Boeing/Bob Ferguson.

Welcome

Welcome to our seventh issue of NZIMAGES, containing articles on the meeting point between maths and several natural phenomena including earthquakes, icebergs, waves, wood and weeds. There's also an interview with an aspiring NASA astronaut and a song about algorithms. We hope you enjoy it.

Marston Conder and Vaughan Jones
Co-Directors





See also

Willcox's page - <http://acdl.mit.edu/willcox.html>

The Silent Aircraft site - <http://silentaircraft.org/>

The proposed silent aircraft design.

Below: From computer modelling of astronaut collision constraints.

$$L \equiv \prod_{K=1}^{15} (L_K)^{1/15}$$

◀ **I** fining the mathematical definition of the problem. The maths contribution allowed the team to evaluate thousands of designs overnight rather than manually change one element at a time."

"Mathematical methods are having a big impact on design. In aircraft design, safety considerations have meant that aircraft are heavier than they need to be. Statistical methods have the potential to come up with much more efficient systems and better designs. For example, an on-board computer alleviating wind gusts by wiggling the flaps in real time could save building extra structure into the wing to withstand them."

"Mathematics shows up everywhere, including computer models of how astronauts move in space - matrices, geometry, trigonometry, derivatives. All those things I learnt about in school that seemed abstract and not very practical turned out to be very meaningful."

Willcox isn't fascinated just with mathematics; she has applied for the next NASA astronaut intake after just missing out on the last one. After an eight-day immersion interview in March, she was in the shortlist of 40 out of 4,000 who applied, but budget cuts meant only nine were accepted. Applicants are selected for teamwork, ability under pressure, creativity and analytical skills.

Before she had her new baby, Willcox played on the MIT women's rugby team, competed in marathons and ultra-marathons, and climbed rock faces. "I really enjoy the challenge of going somewhere I haven't been before and putting myself far away from safety", she says.

Her sabbatical places her close to two grandmothers for her first child Pieter, and brings her back to the University of Auckland where she studied after Lincoln Heights primary school and St Cuthbert's College. "There's a lot of stochastic optimisation in Engineering Science at the University of Auckland, so it's a good community for the kinds of things I'm learning about."

NOTABLE MATHS PROBLEMS

NAVIER-STOKES EXISTENCE AND SMOOTHNESS PROBLEMS

That solutions to the motion of fluids in three dimensions always exist (existence); and that if they do exist, then they do not contain any singularity, infinity or discontinuity (smoothness).

Also stated as: Show that the Navier–Stokes equations on Euclidean 3-space have a unique, smooth, finite energy solution for all time greater than or equal to zero, given smooth, divergence-free, initial conditions which decay rapidly at large distances. Or show that there is no such solution.

Discipline: Analysis.

Originators: French mathematician Claude-Louis Navier and English mathematician George Gabriel Stokes in 1822.

Incentive: US\$1million, one of the seven Millennium Prize Problems - the most important open problems in mathematics, according to the USA-based Clay Mathematics Institute.

Usefulness: The Navier-Stokes equations are nonlinear partial differential equations in almost every real situation. They describe the physics of weather, ocean currents, water flow in a pipe, air flow around a wing, and the motion of stars in a galaxy as well as help with design for aircraft, cars and power stations, the study of blood flow, and pollution analysis.

Explorations: One approach - constructing a weak solution and showing that any weak solution is smooth - has had partial success. It is believed, though not known with certainty, that the Navier-Stokes equations describe turbulence properly. However, the equations are supercritical - energy can interact much more forcefully at fine scales than it can at coarse scales. There is no good large data global theory for any supercritical equation, without additional constraints.

Almost all the equations are written for Newtonian fluids, which continue to flow regardless of forces acting on them. Models for other kinds of fluid flows, such as blood, do not yet exist.

State of play: Since we don't even know whether solutions exist, our understanding is primitive. Some exact solutions of degenerate cases and non-linear equations do exist. Solutions may lie in related models, such as the Euler equations.

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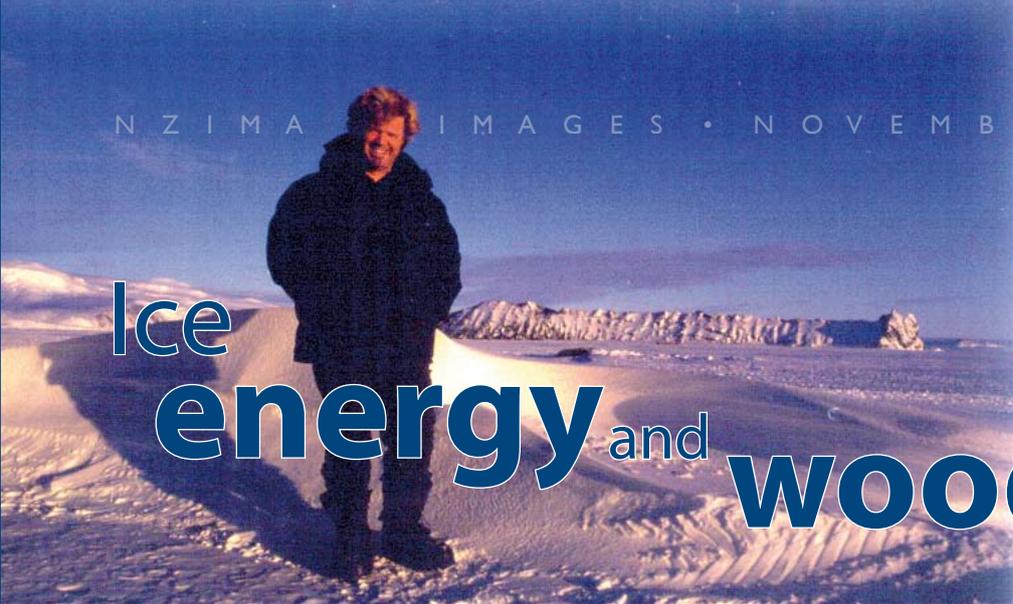
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First appearances can be deceptive. While the ethos of every other science is that experimental evidence is all that you can truly rely on, mathematicians have learnt never to trust numerical data without proof.

Marcus de Sautoy, Music of the Primes.

Ice energy and wood



“The magic of mathematics is that the same methods have applications in a whole range of fields,” says Colin Fox. He described some to Jenny Rankine.

“Partial differential equations (PDEs) model energy propagating through a system that varies in space and time, like sound or seismic waves, or x-rays through human tissue,” says Fox (above).

“For example, the gung-ho overuse of early geothermal fields is no longer acceptable,” he says, “so those fields need to be modelled for environmental management and sustainable power generation.” But underground measurement involves a lot of signal noise and uncertainty.

Fox was a co-director of the NZIMA programme on Analysis, Applications, and Inverse Problems in PDEs in 2007, and with co-director Professor Mike O’Sullivan, supervised NZIMA scholarship student Tiangang Cui to compute a Markov Chain Monte Carlo method.

“But if you use basic MCMC methods, the universe wouldn’t last long enough for the computer programme to finish, so we developed some algorithms around that,” says Fox. “The computations then became faster than standard engineering least squares optimisation methods; it’s like a big camera for looking at the geological structures.”

The solution is now used for resource consent hearings and long-range planning. Auckland was one of three centres in the world where the same combination of PDEs from engineering, inversion in graduate statistics and numerical and analytical maths was taught. The notes for Physics 707 - Inverse Problems, written in 1997 by Fox with colleagues Geoff Nicholls and Sze Tan, reached the 60 most-downloaded mathematical texts online. The area became a priority for the USA National Science Foundation this year, so Fox predicts that they’ll catch up soon.

The largest seasonal process in the world, the southern ice freeze and thaw, is a very mathematical system, says Fox, and was another focus for his work with PDEs for more than 12 years. In the 1990s, he was part of the New Zealand science team K131, which studied sea ice far from Scott Base.

“The sea ice freeze was the biggest effect in southern hemisphere climate models, but modellers didn’t know how to include it,” he said. No one had made the measurements Fox needed to describe how ocean waves affect land-fast sea ice, so for years his team tried to gauge them with sensitive tilt meters at the edge of the sea ice, “hoping it didn’t break that day and float off”.

When they decided to make their own waves, the team built a hydraulic jack they called the Thumper. “In the first year, I left out some terms in the residue late one night, which cost the taxpayer about \$50,000 because the Thumper was about ten times too small.”

The second year he recalculated, and the 3m by 3m jack happily created waves by picking up two tonne lumps of ice and dropping them, safely away from the edge. The results led to better mathematical methods for solving those kinds of problems.

“To a mathematician, sea ice is a combination of fluids, thin elastic plates and waves. Lightweight timber construction in New Zealand is similar - plates and beams in a fluid.” Fox had been interested in acoustics as a student, and directed the Acoustics Research Centre from 1998 to 2007.

“Sound insulation in houses is a complicated problem - over the years we saw hundreds of entrepreneurs with the latest idea for quiet walls.” Fox supervised post-doctoral student Hyuck Chung, who wrote the codes and did the modeling to design a timber floor with better sound isolation than concrete.

“We used a version of finite element methods, a standard engineering technique for solving PDEs. We wouldn’t have come to these methods without the semi-analytic methods we’d developed for sea ice.” They worked with acoustics and building specialists, building a simple floor and making sure the

computer model exactly matched.

“Two designs both performed better than concrete floors. They included careful placement of joists, choice of material and layering - nothing exotic. One included a novel element - a layer of mixed sand and sawdust between ply and floorboards.”

“Listener tests were part of final testing. We recorded the noise of walking over the floors and replayed them in a listening room. You would swear that someone was walking on the floor above you, and we asked them to rate the noise as better or worse. That’s why I enjoy acoustics - at the end the human ear is the final arbiter.”

**Below: The thumper.
Bottom: The sand and sawdust sound isolating floor.**

