Infinity without a repeat

The Penrose Patterns activity during Incredible Science day at the University of Auckland in July involved 2,000 kite and arrowhead shaped perspex tiles in five colours.

With these two simple shapes, primary and intermediate students could create kings and queens, suns, stars, birds and worms, and learn why some patterns could go on infinitely without ever repeating.

Hugh Gribben had the idea and got the tiles made. Each tile had a hole in one corner and there were two simple rules - the corners with holes had to go next to another corner with a hole, and the long edges had to go next to other long edges. Hundreds of students took part, most starting their own small patterns separately all over the room. "Kids who could hardly speak could put tiles together," said Dr Isabel Hubard, who ran the project. "But at one point we got stuck, because it can be impossible to join lots of small groups of tiles." Hubard changed the colours on one section with lots of symmetries and told the children how to resolve the problem. "Then it was big enough that people could add tiles all around the edge and not get stuck."

Photos: Tiger Tiger.

Playing with polytopes

Like many mathematicians working with symmetries, Dr Isabel Hubard started working with polyhedra because she loved their beautiful shapes. "I like to draw pictures, use different colours and study them for hours. Playing like a five-year-old, I can see which mathematical properties they satisfy." Hubard prints out her own triangular and hexagonal graph paper for doodling.

Hubard, who hails from Mexico City, touched down in Aotearoa for 2008, after a Masters and PhD in York University in Toronto, and before a post-doctoral position in Brussels, Belgium. She is a temporary lecturer at the University of Auckland in graph theory.

Mathematics orders the relationships of vertices to edges and faces in polyhedra. "A vertex is smaller than an edge if it is one of the endpoints; an edge is smaller than a face if it belongs to the face. Then we can forget about what they look like physically and think of them as objects with an order between them. We can use as many layers or ranks as we like. We say the faces are of rank 2 to any finite number; they are abstract polytopes, unable to be built in the 3-space we occupy."

Regular polyhedra - such as tetrahedra, cubes and octahedra - are the most studied; they include many symmetrical reflections and rotations. Other polytopes that have all the possible rotations, but not reflections, are called chiral. A related vertex, edge and face on all polytopes is called a flag.

"When polytopes are regular, the number of symmetries they have is the same as the number of flags," says Hubard. "And the reverse is true for finite polytopes – if the number of flags is the same as the symmetries, then the polytope is regular."

"For finite chiral polytopes, the number of symmetries is half the number of flags. But the reverse doesn't work - if you have half the symmetries, it doesn't mean that it's chiral." Hubard studied two-orbit polytopes for her PhD.

Symmetries can divide the flags into two sets, called orbits. "Given a two-orbit polyhedron, we can tell the group of symmetries, with its generators and some relations they will satisfy. If you give me any symmetrical group, with generators and relations, I can tell you if it will be a group of two-orbit polyhedra and if it is, I can construct it."

She also worked on what happens to polyhedra when the order of vertices, edges and faces for each shape is reversed. "When we reverse the order, we get a different polytope. A cube becomes an octahedron and a dodecahedron becomes an iscosahedron. But a tetrahedron becomes a tetrahedron in a different place; this is called self-dual."

"It is intuitive to think that if we reverse the order twice, we get the same object. The problem is that the same object may end up in a different position." After two reversals, regular polytopes can always to return to the same position; those with very little symmetry don't always return to the same position. Hubard and her supervisor Asia Weiss found that chiral polytopes of odd ranks can always come back to their initial position, but that some of those of rank 4 cannot. Later, with Alen OrbaniÐ, Hubard and Weiss found a rule that decided for any polytope.

She is now working on generalising from two-orbit polytopes to those of any rank, with Professor Egon Schulte at North Eastern University in Boston. And when she's not doing maths, she's teaching tango in downtown Auckland.

MATHEMATICAL EVENTS

24-25 November, Wellington
Operations Research Society of NZ 43rd Annual Conference
http://conference.orsnz.org.nz

7 December, Christchurch
Half-day Workshop for Women Researchers in the Mathematical Sciences in NZ
Contact Vivien Kirk, v.kirk@auckland.ac.nz

8-12 December, Christchurch
7th Australia-NZ Mathematics Convention (incorporating the 2008 NZ Mathematics Colloquium)
www.math.canterbury.ac.nz/ANZMC2008

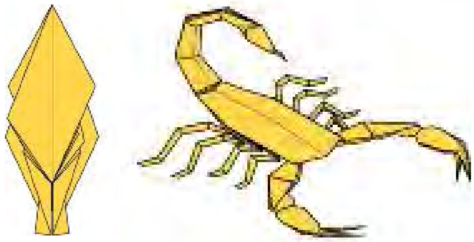
15-19 December, Auckland
4th International Conference on Combinatorial Mathematics and Combinatorial Computing
www.cs.auckland.ac.nz/research/groups/theory/4ICC/index.html

4-9 January 2009, Napier
Annual NZMRI/NZIMA Summer Meeting, on algorithmic information theory, computability and complexity
www.mcs.vuw.ac.nz/Events/NZMRI2009/WebHome

1-5 February, Caloundra, Queensland
ANZIAM 2009 (annual meeting of Australia & NZ Applied Mathematics)
www.sci.usq.edu.au/conference/index.php/ANZIAM/2009

6-10 July, Sydney, Australia
First Pacific Rim Mathematical Congress
www.primath.org/prima2009/

29 September - 2 October, Palmerston North
Pi in the Sky: Extending Mathematical Horizons, Biennial Conference of the NZ Association of Mathematics Teachers (NZAMT11)
www.nzamt.org.nz/nzamt11/



Learning by **folding**

Don't be surprised to see an origami workshop advertised at a high school near you in 2009. New Zealand origami master Jonathan Baxter and Auckland mathematician Hugh Gribben, pictured, took their Great Origami Maths and Science Show on a highly popular national tour in 2006, and they're keen to repeat it. By Jenny Rankine.

Gribben folds coloured pieces of paper as he speaks, creating elegant shapes or modular polyhedra. But folding and talking don't go well together unless he practises, and after a conversation his desk is littered with finished and half-finished shapes, intriguingly creased.

He describes the tour as a "whizz-bang show, not a workshop", which used origami to turn kids onto maths and their teachers onto its ability to illustrate tangibly many maths areas. The show also explored science principles, such as aerodynamics with paper darts and how DNA helixes collapse.

Students were bussed in to main centres for the show, which had booked out audiences of up to 300. The pair folded on stage, with live close ups on a big screen, and invited some students on stage to fold particular shapes. Each class received a copy of the 150-page book of the show.

Gribben has used origami as a first year maths guest lecturer, and is regularly invited to run school workshops. "It is tangible rather than abstract learning; the kids learn it with their hands." He may start by asking them how many ways they can fold a square of paper into a new shape that is half the area of the original square.

His favourite example was first documented in 1893 by T. Sundara Row and starts with all four corners meeting in the middle of the square. The new square is half the area of the original. In Row's book, this is repeated until the square is too small to continue.

Imagining the folding going on for ever provides a proof of the convergence of the geometric series. Gribben says students find this geometrical proof very convincing when they fold it themselves.

Another exercise can involve making a nautilus shell similar to the NZIMA logo, or a self-similar wave. "If I did that with 100 steps, they would all have the same ratio between the folds of the wave front." Year 13 maths students can then use complex numbers to find the centre of the spiral.

He also gets students exploring Platonic or Archimedean solids. Modular polyhedra are his favourite - "they're simple shapes put together - lots of eye candy". Gribben is pictured making an alpha prism. This semi-regular solid is made of six modules, each a paper square folded three times into a right isosceles triangular shape.

He likes to make them with three paper colours. "I make it so different colours don't lie next to each other, don't share a common edge and the outside is made up of four small triangles. These are aesthetic decisions, but they're also strongly mathematical. To make two the same is difficult - it teaches precision of thinking. It's a very simple but deeply meaningful shape."

Another exercise involves conic sections. "If students mark a point near the bottom of an A4 portrait page and fold different points of the bottom edge to that point, they get a family of folds that are all tangents to a parabola. They can do similar things with an ellipse and a hyperbola."

Famous ancient problems such as doubling a cube and trisecting an arbitrary angle, which the Greeks attempted unsuccessfully to solve with compass and straight edge, can be solved with origami. "I like to believe I could come up with a workshop about any mathematical area," says Gribben.

Computational origami has solved engineering and science problems, such as folding and opening space telescopes and solar panels on satellites. "Studying paper crumpling is an easy way to learn how to model bumpers crumpling during car crashes, or plane bodies on impact," he says.

"Airbags are another application. They have to unfold very rapidly without hard edges." The solution used algorithms developed by Robert Lang for his Treemaker origami programme. "A stent, which has to move along blood vessels then open up and lock into place, uses a very simple origami technology."

Once Gribben has finished maths studies for a Postgraduate Diploma in Science he wants to be "an itinerant maths teacher, running workshops, doing tangible learning". Watch out for Gribben and Baxter in 2009.

Top: A folded base, and the finished scorpion folded from that base.

Background: Screen shot of the computed crease pattern for the scorpion using TreeMaker 4. Circles correspond to terminal flaps.

See also

The Great Origami Maths and Science Show book, available for $27 from Origami New Zealand, c/- Rotorua Arts Village Experience, 1240 Hinemaru St, Rotorua, phone 07 348 9008, fax 07 343 7108, email jbax@mindspring.com

The GOMSS website - www.nzamt.org.nz/origami.htm

A talk by Robert Lang - www.ted.com/index.php/talks/robert_lang_folds_way_new_origami.html

Robert Lang's website - www.langorigami.com/science/science.php4