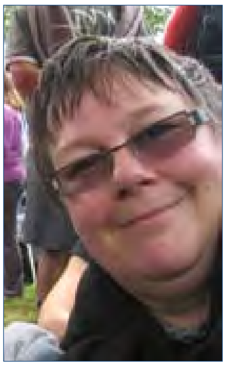
Maths in psychology



When Sue Street moved from engineering to psychology at Massey University, she brought with her an unusual approach for this people-focused social science – mathematical simulation of the interactions of autonomous individuals that calculates their effects on a system.

In her Masters, she used this agent-based modelling to explore the impact of income inequalities on people's relationships; in her PhD, she modelled development of trust in people's behaviour on the internet auction site Trade Me.

"As an engineer coming into psychology, I was struck by the relative absence of a dimension that is always present in electrical engineering: time. And when you think about time, you're thinking about dynamics. Apart from certain fields like developmental psychology, time is curiously absent in psychology."

For her masters, Street wanted to explore the impact of differences in income, which research has shown has much more impact on the health and mortality of populations than absolute levels of poverty or wealth.

"If two people grow up in Taita and one becomes a millionaire, do they stay friends? I wondered whether the increasing income differences strain existing relationships, and cause a cascading breakdown in relationships within a population. People tend to be friends with those who are similar, so increasing the difference in income is likely to reduce that similarity."

She modelled friendships rather than marriages or sexual relationships, in a finite element structural simulation. She randomly allocated difference in wealth at two relating nodes, and how much time and effort (loading) each node put on the relationship.

"Reducing the similarity in some relationships caused some to break, and shifted the loading on other relationships, some of which also broke, so there were fewer relationships in the population. Reversing the differences didn't recover the overall level of activity in relationships, which implies that closing the gaps in the real world might not work by itself – we'd have to do something to rebuild the effects of prolonged inequality."

For her PhD, Street used Repast, which is free open-source agent-based modelling software, to create a group of traders with a randomly-assigned strategy for dealing with the limited information that Trade Me supplies. This includes the person's number of trades, their trade rating, whether or not individual trades were successful and the reasons why some were not. This could be because someone didn't respond, didn't pay or didn't send the goods. Some agents were randomly more likely to be dishonest and all could learn by getting random strategy information from traders who were doing better.

Street found that agents "got better at identifying and avoiding unreliable traders, but that learning the signs of dishonest traders and avoiding them was patchy at best. Overall, agents improved their trading success, and restricted unreliable traders, but left dishonest traders in circulation." Dishonest traders tend to re-enter Trade Me with a new ID once their original rating deteriorates, but she did not build that into her model.

Street believes that mathematical simulations are a valuable approach that is underused in psychology for analysing very complex patterns of social behaviour. ***JR***

Notable maths problems

BIRCH AND SWINNERTON-DYER CONJECTURE

Simply: Describing solutions to algebraic equations like $x^2 + y^2 = z^2$ in whole numbers becomes extremely difficult for more complicated equations. This conjecture says that for an elliptic curve (a three-dimensional curve confined to a region known as a torus), the number of solutions depends on the behaviour of an associated function (the zeta function).

Originators: British mathematicians Peter Swinnerton-Dyer and Bryan Birch in the early 1960s.

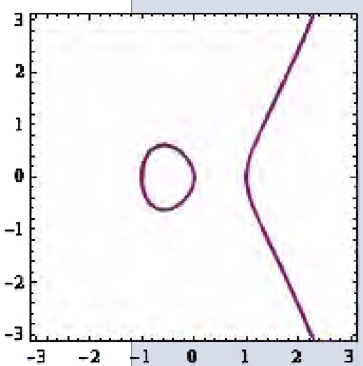
Discipline: Number theory.

Incentive: $US1million for the first proof of the whole conjecture; one of the seven Millennium Prize Problems of the USA-based Clay Mathematics Institute.

Interesting aspects: The conjecture "relates the behavior of a function *L* at a point where it is not at present known to be defined to the order of a group which is not known to be finite", according to mathematician John Tate. Elliptic curves from algebraic geometry are important to number theory – proving statements about these curves was crucial in solving Fermat's famous last theorem in 1995.

Progress: Proofs of the conjecture have been developed from 1976 to 2010 but only in special cases, all with rank less than 2. Nothing has been proved for curves with rank greater than 1, although there are many numerical calculations supporting its possible truth. The conjecture has stimulated a lot of research and is widely recognised as one of the most challenging unsolved mathematical problems. Many details about it have become clear only after hard computation, one case at a time.

NZIMA connection: Bryan Birch was 'moral tutor' for NZIMA Co-director Marston Conder when Marston was a doctoral student in Oxford in the late 1970s.



Elliptic curve $x^3 - x = y^2$

Speeding medical images

NZIMA Scholar Rachael Tappenden is studying the algorithms behind Magnetic Resonance Imaging (MRI).

MRI scans are one of the most important diagnostic techniques that doctors can use, but it is very expensive technology, partly due to the time it takes to produce and analyse such complex images. Mathematics provides the central algorithms for analysing and reconstructing the images in real time.

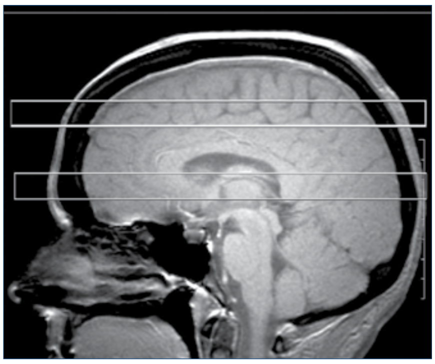
The two common algorithms, Sequential Backward Selection and Sequential Forward Selection, force a compromise between analytical speed and image quality.

Tappenden and supervisor, Associate Professor Ian Coope at the University of Canterbury, chose this issue for her PhD as "the most interesting that uses the skill base I have – linear algebra and optimisation," she says.

The algorithms rely on Least Squares approximations and the L^2 norm, but other approximation criteria, like the L^1 norm, may be useful. The images are compressed before processing using Fourier methods, but recent techniques such as compressed sensing are also being explored as an alternative.

MRI data comes in the form of a matrix but it is not feasible to look at every combination of rows. "An existing criterion uses the trace of the matrix," says Tappenden, "whereas we've created an algorithm which uses the determinant criterion to choose an optimal subset of rows to give an accurate image." Her paper was published in September in the IEEE journal Transactions on Image Processing. The algorithms will be implemented by the engineers who programme the machines.

Tappenden has also written some algorithms to reconstruct MRI scans from sparse data sets. "If you have some conditions on the image – lots of zeros and few non-zeroes – then you can do a really good reconstruction with only a tiny bit of data. These are optimisation problems with some really nice properties." ***JR***



Brain scan showing the location of two 15-minute slices of MRI data taken at different levels.

$$\frac{a^H(A^HA)^{-2}a}{1-a^H(A^HA)^{-1}a}$$

"Mathematical reasoning may be regarded as the exercise of a combination of two facilities, intuition and ingenuity."

Alan Turing, 1912-1954

MATHEMATICAL EVENTS

3-7 July 2011, Alice Springs, Australia
34th Annual Conference of MERGA
(Mathematics Education Research Group Australasia)
www.aamt.edu.au/index.php/Conferences/AAMT-MERGA-conference

28-31 August 2011, University of Auckland
Annual Conference of the NZ Statistical Association
www.stat.auckland.ac.nz/nzsa2011/

27 November - 2 December 2011, Rotorua
Volcanic DELTA: The 8th Southern Hemisphere Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics
www.delta2011.co.nz/delta2011

6-8 December 2011, University of Auckland
2011 New Zealand Mathematics Colloquium
www.math.auckland.ac.nz/NZMC2011

15-20 December 2011, Victoria University of Wellington
The 12th Asian Logic Conference
http://msor.victoria.ac.nz/Events/ALC2011/WebHome

AWARDS AND HONOURS

LEN COOK (NZIMA Board Chair) has been elected a Life Member of the NZ Statistical Association.

SHAUN HENDY (director of the NZIMA programme on Applications of Mathematics in the Nanosciences) has won the 2010 Research Medal of the NZ Association of Scientists.

ANDRÉ NIES (a member of the NZIMA Logic and Computation programme committee) was elected a Fellow of the Royal Society of NZ in October.

CHARLES SEMPLE (co-director of the NZIMA programme in Algorithmics) won the NZ Mathematical Society's annual Research Award for 2010.

RACHAEL TAPPENDEN (an NZIMA scholar) won the Aitken Prize of the NZ Mathematical Society for 2010.

GEOFF WHITTLE (an NZIMA PI and a former Maclaurin Fellow) has been selected by the London Mathematical Society and NZ Mathematical Society as its first Aitken Lecturer.

Luminiferous aether

The application of calculus in physics may not sound like a subject of public entertainment, but the success of a pilot theatre show about it has led to the development of a full-length New Zealand play.

Professor Matt Visser and Nick Wyatt were the mathematics and physics advisors in a collaboration with Wellington's Lumina Productions, above, that led to the pilot show *The Amazing Adventures of Doctor Faustroll and his Search for the Luminiferous Aether* in 2010. Lumina included producers Mark Westerby and Howard Taylor, director Charlie Bleakley, designer Joe Bleakley, writer Jean Betts and composers John Psathas and David Downes.

The 20-minute selection of scenes aimed to bridge the gap between quantum physics and the stage. It was funded by the Ministry of Research, Science and Technology's Smash Palace Fund, which has since funded script development for a full-length play.

"It's not finalised yet," says Visser. "We're discussing calculus, waves, basic physics ideas as well as quantum physics. In the pilot we discussed Olbers' Paradox, which involves geometry and the inverse square law." It argues that if the universe is infinite, at any angle from the Earth the sight line will end at the surface of a star, so the night sky should be completely white. The darkness of the night sky is one piece of evidence for the Big Bang model of a non-static universe.

"Some people got it, and some went away puzzled," says Visser. Full-length script discussions currently include the Pauli Exclusion Principle from quantum physics, "which says thou shalt not have two electrons in the same place at the same time. It is impossible to squeeze two atoms into the same spot at all, and difficult to move them closer than normal without dynamite or a nuclear explosion."

The script will be finished by the end of the year, and Lumina aims to tour the play.

See
www.htproductions.co.nz/Faustroll.htm

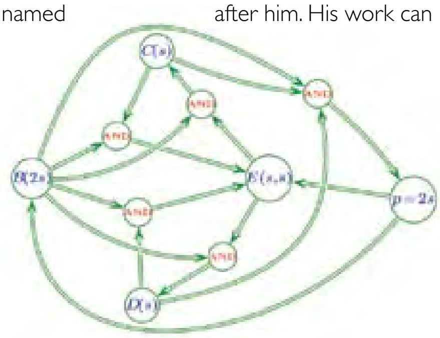
First Jones medal

John Butcher was awarded the inaugural Jones medal, named after NZIMA Co-director Vaughan Jones, by the Royal Society of New Zealand in November, and the Van Wijngaarden Award by the Centrum Wiskunde & Informatica in Amsterdam in February.

The Jones Medal is for lifetime achievement in pure or applied mathematics or statistics, and is accompanied by a $5,000 prize from the NZ Mathematics Research Institute. These awards recognise Butcher's exceptional work on numerical methods for solving differential equations, and the Jones Medal also recognises his leadership contributions in New Zealand.

Differential equations are used to study the motion of objects acted on by forces. Butcher used rooted trees, which come from a different area of mathematics, to study solutions of particular differential equations by the Runge–Kutta method. Formulae for the series of exact and approximate solutions can be written in a series of rooted trees. In the process he constructed an infinite-dimensional group that has now been named after him. His work can also be applied in the simulation of waves.

Butcher was Professor of Mathematics at the University of Auckland from 1966 to 1998, and founded its Department of Computer Science. He is now an Emeritus Professor and continues his research at a very high level.



MATHEMATICAL JOBS TOP-RATED

Six of America's ten best jobs in 2011 require a strong background in mathematical science.

The annual Jobs Rated report ranked software engineer as the top job, followed by mathematician, actuary (employed by insurance companies to determine the probabilities of accidents, sickness, death and property loss from statistics), statistician, computer systems analyst and meteorologist. The top income was US$94,000 for mathematicians.

The report rated 200 occupations for work environment, physical demands, outlook, income and stress, and the best jobs scored high in all or most of these fields.

On average, these professions offer better than average income, comparatively low stress, a comfortable work environment, few intense physical demands and strong hiring. In short, they are much more satisfying.

See www.careercast.com/jobs-rated/10-best-jobs-2011