Amir Joshan and Andy Philpott. Photo: Godfrey Boehnke.

# Revenue management

**By Jenny Rankine**

You're enduring the Auckland to Los Angeles leg of your long-awaited OE, looking forward to landing in London in 20 hours. Your flights were booked months ago at the lowest rate you could find. The passenger on your left got her ticket yesterday for a conference in San Francisco tomorrow and is working frantically on her presentation. The guy on the right bought his single-leg ticket for a lower price than yours in an airpoints promotion that started a month after you'd paid for your seat.

You don't spare a thought for the airline staff who have to juggle such unconnected and competing demands. NZIMA programme co-director Professor Andy Philpott, PhD student Amir Joshan and honours student Michael Frankovich, however, do. They are developing more sophisticated revenue management models for airlines. This fast-growing area of mathematics was kick-started in the 1980s with the arrival of the first cut-price airline in the USA. It now encompasses hotel, rental car, sports-event seating and electricity markets.

Airline revenue management models are based on the fact that different customers are willing to pay different amounts for the same seat; the models aim to extract greater income from airplane seating by differentiating between these customers.

"It's difficult mathematics because customers don't show up all at once", says Philpott. "The first complication is deciding how many seats should be reserved for possible high fare customers who haven't booked. The second complication is that most itineraries consist of sequences of flights. The airline has to decide whether it is better to accept a single-leg customer or to keep seats free for passengers wanting a longer route."

"The third complication is competition. These decisions are all affected by how other airlines are pricing their seats." As part of his PhD thesis, Amir Joshan is developing a preliminary model that accounts for competition between airlines, using data from Air New Zealand.

"Most airlines use commercial revenue management systems, developed by USA software companies such as Sabre and PROS," says Philpott. They are generally based on heuristics - rules of thumb that perform well in practice but are not necessarily optimal.

Philpott is also working with Garrett Van Ryzin at Columbia University on improving computation methods for solving network revenue management problems. Frankovich is experimenting with different networks, classes of arrival process, during high and low use of an airline. Philpott will meet Van Ryzin in Veinna in August to complete the project.

"We're using the theory of multi-stage stochastic linear programming to develop policies that can be proved to be within a certain tolerance of optimal, but we're still a fair distance from the real picture," says Philpott.

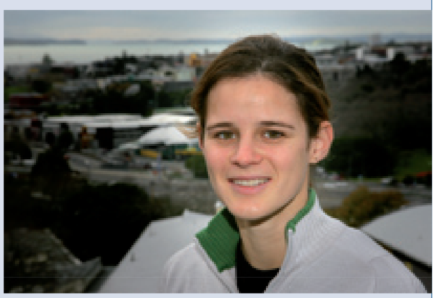
"We sample from an idealised model of arrivals that is based on historical airline data. We develop a policy about whether or not to accept a customer requesting a certain type of seat, and then simulate it in comparison with the deterministic linear programming policies that are typically used in practice. In preliminary experiments our policies are doing better."

$$\text{maximize } r^T y \text{ subject to } Ay \leq x,\ y \leq E[D],\ y \geq 0.$$

## Postgraduate scholarships

**NZIMA awarded three new Postgraduate Scholarships in 2006. These went to Dion O'Neale at Massey University, and Tiangang Cui and Kim Noakes at the University of Auckland.**

### Pelvic floor modelling



Masters' student **Kim Noakes** is working to create anatomically accurate and patient-specific computational models of the male and female pelvic floor and anal canal regions. Ultimately this will aid the diagnosis of pelvic floor disorders and enable virtual planning of corrective surgeries. These disorders seriously reduce the quality of life of up to 15 percent of the population. Noakes has created one patient model, and hopes to use it to perform a functional analysis of a muscle contraction. The same MRI patient has undergone a manometry study to record the pressures in the anal canal exerted by their surrounding muscles. Noakes will try to match this mathematically on the computer model of the patient.

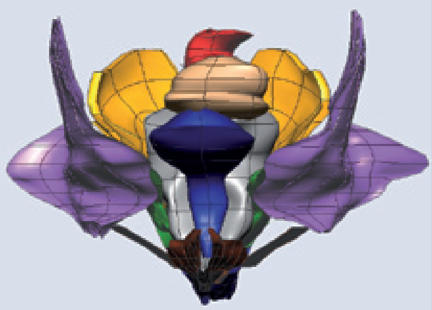
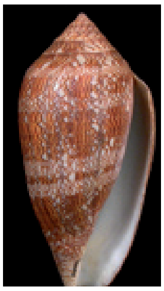


Photo: Godfrey Boehnke. The pelvic floor model is shown from the front.

# Varied applications of cell grids

**Most non-mathematicians would be struggling to make meaningful connections between forest fires, earthquakes and urban gang recruitment. Not so William Newman. He spoke with Jenny Rankine.**



Newman, a professor from the University of California at Los Angeles, is an NZIMA-sponsored visiting expert who lectured on cellular automata and chaos theory in Auckland and in Christchurch in July, and renewed connections with New Zealand mathematicians.

Cellular automata arose from mathematical models of biological systems and descriptions of natural and engineering networks. It consists of a large, regular grid of cells, each in one of several states, modelled over time. Its roots are in formulae for object permutations and combinations in systems, fractal geometry and directed graphs.

To apply the discipline to forest fires, a mathematician assumes tree seeds are planted randomly in an area at a given time. New seeds will not take root where another tree is growing, leading to patterned clusters of mature trees. To stimulate fires, the mathematician imagines a random hand dropping matches or lightning bolts.

"The accumulation of flammable material on forest floor sets up clusters of trees for calamitous fires," says Newman. "In this imaginary landscape a cluster will be destroyed if lightning hits a tree. Eventually, equilibrium emerges between clusters of trees and fires. The model can suggest the best time and place for limited controlled fires to eliminate underbrush so that any accidental fires are self-limiting." This work has been of great interest to the US Forest Service.

Cellular automata are also applied to the distribution and size of earthquakes, which release accumulated stress from deformations in the earth's crust. Newman says that many years ago, a UK meteorologist became interested in the use of maths to address conflict. "He gathered data on gangs from Chicago in the 1920s and in occupied Manchuria in the early 1940s. The stats plots exactly matched what we see in forest fires and the statistical distribution followed the power law, for example, describing how much more common smaller gangs were than those twice as big."

"Planting a tree is equivalent to recruiting a gang member. A forest fire is equivalent to the break up of a gang - the common feature is geometry, their relationship in space." Newman's research has led him to see gangs as a universal response to urban social problems, unrelated to any particular ethnicity or time.

"If our model has any sense, the place we can help prevent recruitment is in areas with very few gang members. We won't get very far where they are entrenched. Of course," he qualifies, "this is an extrapolation with no scientific basis as yet."

During his visit, Newman renewed his friendship with Professor James Sneyd at the University of Auckland, who was involved with the 2003 NZIMA programme on modelling cellular function. They are both now working on the mathematics of brain tumours and Newman believes cellular automata may provide a useful approach.

**William Newman. Photo: Godfrey Boehnke.**

"Cells don't just sit there - they reproduce, die and multiply. But maths methods about the behaviour of fluids are based on the notion that molecules of water are neither created nor destroyed. Cellular automata could provide a way to account for this."

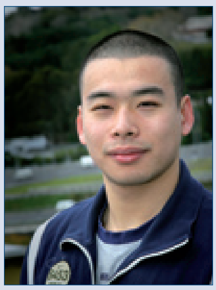
## Postgraduate scholarship

### Geothermal modelling

PhD student **Tiangang Cui** is exploring statistical techniques for quantifying the parameters and modelling errors associated with numerical models of geothermal fields.

These models simulate multi-phase flow - the simultaneous movement of gases, liquids and solids - by solving a large system of non-linear partial differential equations (PDEs). PDEs relate an unknown function of several independent variables and its partial derivatives for those variables. As well as the flow of fluids, PDEs are used to formulate and solve problems such as the propagation of sound or heat, electro-dynamics and elasticity.

Geothermal field models use finite volume methods, which represent and evaluate PDEs as algebraic equations. Values are calculated at discrete places on a meshed geometry; finite volume refers to the small volume surrounding each node point on a mesh.

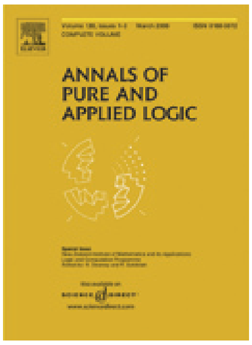


Cui is researching sample-based Bayesian inference to calibrate the model. This method uses observations from the geothermal fields to calculate the probability that the model may be accurate.

**Photo: Godfrey Boehnke.**

## Journal special issue

A SPECIAL ISSUE of the journal *Annals of Pure and Applied Logic* has been published by Elsevier, as part of the NZIMA programme in Logic and Computation. This appeared in 2006 as Volume 138 of the journal, with guest editors Rod Downey and Rob Goldblatt from Victoria University of Wellington.

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## MATHEMATICAL EVENTS

15 March 2007, University of Auckland
**Music of the Primes**, Public lecture by Professor Marcus du Sautoy

16 March, Victoria University of Wellington
**Why flatland is a great place to do algebra**, Public lecture by Professor Vaughan Jones

16 - 20 April, Hanmer Springs
**NZIMA Programme Workshop on Modelling Invasive Species and Weed Impact** www.math.canterbury.ac.nz/bio/NZIMA/

4 - 6 July, Christchurch
**2007 NZ Statistical Association Conference,** nzsa2007@gmail.com.

25 - 28 September, Auckland
**New Zealand Association of Mathematics Teachers (NZAMT) 10th Biennial Conference,** www.cce.auckland.ac.nz/conferences/index.cfm?S=CCE_NZAMTC

3 - 7 December, Dunedin (to be confirmed)
**32nd ACCMCC (Australasian Conference on Combinatorial Mathematics and Combinatorial Computing),** mike@cs.otago.ac.nz

12 - 15 December, Victoria University of Wellington
**1st Joint Meeting of the American and New Zealand Mathematical Societies,** www.mcs.vuw.ac.nz/~mathmeet/amsnzms2007/

## Awards and honours

**PROFESSOR ANDY PHILPOTT** was presented with the 2006 Daellenbach Prize in November for his major contributions to the theory and practice of operations research in New Zealand and internationally. Philpott is Co-Director of the NZIMA programme on Mathematical Models for Optimizing Transportation Services, and Head of the Department of Engineering Science at the University of Auckland.

The three-year prize was established by the Operations Research Society of New Zealand (ORSNZ) to honour the contributions of Emeritus Professor Hans Daellenbach and reward outstanding examples of management science and operations research. The citation acknowledged Philpott's publications in prestigious international optimization and operations research journals and his many invitations to present at major international conferences.

**ORSNZ President, Professor David Ryan, left, with Professor Philpott at the Royal Society of New Zealand awards night. Both are NZIMA Board Members. Photo: Andrew Mason.**

**ROD DOWNEY**, the first NZIMA Maclaurin Fellow, was an invited speaker at the International Congress of Mathematics, ICM2006, held in August in Madrid, Spain. This is the first time ever that a New Zealand based-mathematician has been invited to speak at the ICM.

**PETER HUNTER**, member of the NZIMA Governing Board and Director of the Bioengineering Institute, and Jerry Marsden, member of the NZIMA International Scientific Advisory Board, have been elected as Fellows of the Royal Society of London.

**CATHERINE McCARTIN**, who was involved in the NZIMA's Logic and Computation programme, has won the Royal Society of New Zealand's Hatherton Award for 2006, for the best scientific paper by a PhD student at any New Zealand university.

**ALASTAIR SCOTT**, member of the NZIMA Board, has won the Waksberg Award for 2006 from the American Statistical Association and the Statistical Society of Canada, for his work on survey sampling.

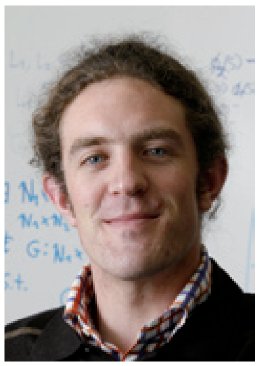
## Postgraduate scholarship

### Applications of geometric numerical integration

PhD candidate **Dion O'Neale** is studying the application of geometric numerical integration to systems of Hamiltonian differential equations. Geometric numerical integrators are computational methods for finding approximate solutions to differential equations. They differ from traditional numerical integration methods in that they attempt to preserve the equations' geometric properties, which often arise from physical laws such as conservation of energy.

One particular application is to coupled-spin systems. This type of Hamiltonian system is widely used in physics for modeling strongly correlated systems. One important feature of Hamiltonian systems is the existence of periodic solutions. When such a system is discretised by applying a numerical integrator to it, as many as possible of these periodic solutions should be preserved. For the discretised system, the analogue of a periodic solution is an invariant circle.

O'Neale is comparing the set of periodic orbits of a Hamiltonian system with the set of invariant circles obtained by discretising the system with a type of geometric numerical integrator. He aims to identify a set of periodic orbits that is always preserved by the integrator.



**Dion O'Neale. Photo: Graeme Brown**