NZ IMAges
ISSUE SIX

INSIDE

New Zealand Institute of Mathematics & its Applications
taurangi

Fastest across the Atlantic

The release of Atlantic rower Kevin Biggar's book late last year made public the link between mathematical optimisation and his and rowing partner Jamie Fitzgerald's world-beating record in the 2003 Trans-Atlantic Rowing Race. Jenny Rankine explains.

In *The Oarsome Adventures of a Fat Boy Rower (How I went from couch potato to Atlantic rowing race winner)*, Biggar describes how his need to find out the fastest route between the Canary Islands and Barbados led to his meeting with two mathematical optimisation experts.

Professor Andy Philpott at the University of Auckland School of Engineering and Auckland consultant Dr Geoff Leyland told him that when the weather is uncertain, the straightest route may not be the fastest.

"What you want is a policy that adapts to the weather," Philpott said. Rowers were not allowed to use routing advice from off the boat during the race, so the pair developed an isochrone map based on 20 years of mean wind patterns across the ocean.

Isochrone means equal time, and each line represented a series of places estimated to be the same time away from the finish if the rowers followed the optimal policy from that point. Such maps start at the race destination and are calculated backwards, with each line representing one day's rowing.

Leyland wrote the code that computed the map as a completely new dynamic optimisation problem using a sample-based version of Bellman's equation for dynamic programming.

He digitised mean wind directions and entered them into a data file of location and probability. "You pick a point and run through 1,000 possible weather probabilities and how long it would take to row," says Leyland. The software calculated the isochrones using the boat speed.

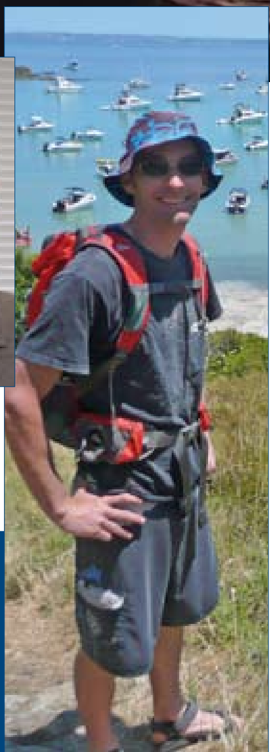
Leyland estimated a speed that challenged the rowers to break the world record, which they did, crossing the ocean in 40 days, five hours and 31 minutes. ►2

Top: Biggar, left, and Fitzgerald after their win; photo: Kenny Rodger, *New Zealand Herald*. Centre: The isochrone map, with Barbados on the left. Above: Biggar on a rowing shift, in a still from video footage taken during the race.

Welcome

Welcome to the sixth issue of IMAges, which contains a range of items on topics including "the Monster", rowing and biofuels. This issue focuses on some of the pure mathematical research being undertaken in New Zealand, especially in algebra. We hope it inspires and intrigues you.

Marston Conder and Vaughan Jones
Co-Directors



Andy Philpott, left, and Geoff Leyland.

◄ In the race, the rowers used the map every shift to determine their rowing direction.

On day 32, when the pair faced headwinds, Biggar describes the map as "invaluable; it gave us the best compromise between the direction of the wind and the next isochrone. Without that, we'd have put the sea anchor out. It was an enormous psychological advantage knowing that, on average, we were going in the right direction."

Philpott says: "The remarkable thing was how accurate the isochrone timing was." Leyland tracked Biggar and Fitzgerald through the race website and was able to counter an appeal by slower boats that the pair had somehow sped up during the race. "The accusation wasn't true; on the isochrone map other teams had slowed down," says Leyland.

Philpott says the optimisation is a great application for long-distance yacht racing. Adds Leyland: "Nothing like that has been done for yacht routing, and nothing since. I still think it's one of the coolest things I've done."

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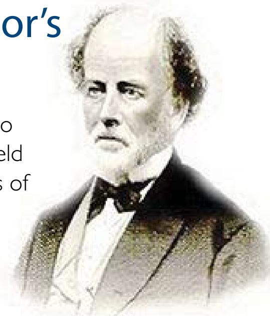
In his ancestor's wake



Leyland was intrigued to use optimisation in a field that built on the efforts of his ancestor, Matthew Fontaine Maury (1806 –1873), an American oceanographer, meteorologist and cartographer.

Maury's *Wind and Current Chart of the North Atlantic* showed sailors how to use the ocean's currents and winds, and drastically reduced the length of ocean voyages. His uniform system of recording oceanographic data was adopted by navies and merchant marines around the world and used to develop charts for all the major trade routes.

Jones in Rome

FESTIVAL DELLA MATEMATICA 2009 CREAZIONI E RICREAZIONI

Vaughan Jones, co-director of the NZIMA, was one of 29 invited international mathematicians and scientists to speak in March at the annual Italian Festival della Matematica, organized by Piergiorgio Odifreddi of the University of Turin and others from Creazioni e Ricreazioni Matematiche.

The previous festival attracted a total of 55,000 people, including the President of Italy, to hear other Fields medalists, Nobel Prize winners and notables in chemistry, physics and economics.

Five hundred people heard Jones's talk, titled *Flatlandia, il luogo ideale per imparare l'algebra,* in the Auditorium Parco della Musica in Rome.

See also -

2009 Festival - http://www.auditorium.com/eventi/festival/4937211
2008 Festival - www.auditorium.com/eventi/podcast?id_podcast=4919070

Discovering the surreal numbers was like discovering a whole new continent. There's a world that no one has seen before. Of course, it's not the same. The surreal numbers are not a physical thing. On the other hand you can carry the concept around in your head, which you can't do with Australia. *John Conway.*

Revelling in abstract maths

University of Auckland mathematician Eamonn O'Brien is making the most of his 2008 NZIMA Maclaurin Fellowship. He spoke with Jenny Rankine.

O'Brien describes himself as "a bit of a butterfly collector; a lot of my work has been on the development of good algorithms for the construction and classification of groups. I've developed techniques to count the number of groups of prime-power order; for example, we can count the groups of the order 1024, 2^{10}. The answer is about 50 billion and it involves a lot of computing."

During his fellowship, O'Brien completed the verification of the long-standing Ore conjecture on finite simple groups with Professor Martin Liebeck of Imperial College, London; Professor Aner Shalev of the Hebrew University of Jerusalem and Professor Pham Tiep at the University of Arizona.

The conjecture, posed in 1951, states that every element of every finite non-abelian simple group is a commutator. "Despite its elegance and simplicity, it has withstood many attacks," he says.

O'Brien also worked with a University of Auckland post-doctoral fellow, Henrik Bäärnhielm, on the development and implementation of Monte Carlo algorithms to construct a chief series for a linear group.

Monte Carlo methods are a class of computational algorithms that rely on repeated random sampling to compute their results; they tend to be used when it is infeasible to compute an exact result with a deterministic algorithm. The chief series algorithm breaks groups into simple building blocks. Knowledge of such a series allows the use of many other algorithms to study the groups.

He and Professor Charles Leedham-Green, from Queen Mary University of London also worked on constructing short presentations for the classical groups of Lie type.

These groups form the central classes of non-abelian finite simple groups. "Such presentations are useful theoretically and practically, particularly in verifying the putative chief series for a linear group," he says.

With Mike Newman from the Australian National University, O'Brien studied the structure of odd order p-groups of fixed coclass.

These are groups that have a power of an odd prime number as their number of elements, with a fixed difference between their composition length and their smallest central number series. With Professor Bettina Eick at the Braunschweig University of Technology and Leedham-Green, they are attempting to understand periodicity among these p-groups, and how it can be used to describe infinite families of groups by a finite diagram or tree.

In May, O'Brien will give a series of lectures to graduate students in China, and in August he will give four invited lectures at the Groups St Andrews Meeting in Bath, the biggest international conference in group theory.

O'Brien describes himself as "very comfortable with abstraction". Despite this, many of his algorithms are part of the basic infrastructure of Magma, a computational algebra system. "People doing computation will frequently use algorithms I'm responsible for, and often for applications or areas I didn't have in mind."

The product replacement algorithm he developed with others in the 1990s has become a standard for mathematicians and statisticians wanting to choose an element reflective of certain properties in large finite groups.

See also

The Magma computational algebra system - http://magma.maths.usyd.edu.au/magma/

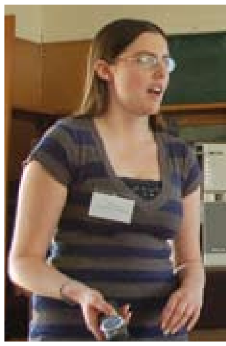
> **"Mathematical reality cannot be located in space or time, [so] it affords - when one is fortunate enough to uncover the minutest portion of it - a sensation of extraordinary pleasure through the feeling of time-lessness that it produces..."**
>
> *Alain Connes, French mathematician*

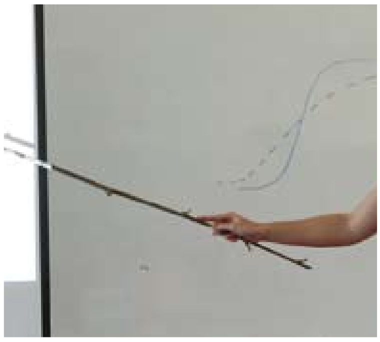
$$\left\langle \begin{pmatrix}1&1\\0&1\end{pmatrix}, \begin{pmatrix}1&0\\1&1\end{pmatrix} \right\rangle = G$$

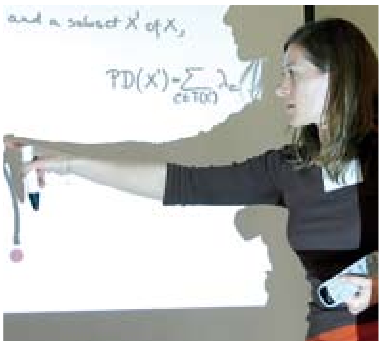
$$|G| = 6$$

NZMASP highlights

More than 60 Mathematics and Statistics students from universities around the country attended the second New Zealand Mathematics and Statistics Postgraduate (NZMASP) Conference in Whitianga on the Coromandel Peninsula in November.

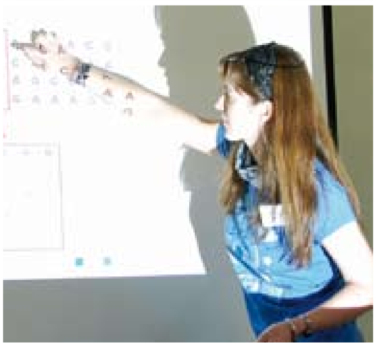


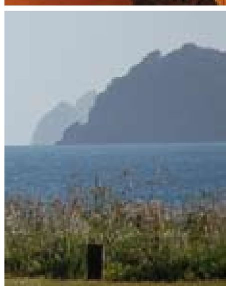




Student talks at the meeting ranged from Representation to Queueing theory, from Phylogenetics to Topology. The quality of presentations was high, with all students managing to explain their work to a varied audience.

The University of Canterbury again ruled the awards; Mareike Fischer's presentation on DNA sequences won the NZIMA Best Presentation Award, while Johnny Humphries' presentation on nesting polynomials earned him the Peoples' Choice Award.





Vertices edges and faces

French-speaking Belgian Alice Devillers, who is based at the University of Western Australia, was collaborating with the NZIMA Co-Director, Professor Marston Conder, during her visit to New Zealand early this year. She spoke with Jenny Rankine.

They were trying to find chiral polytopes in higher dimensional spaces with maximum symmetry. Each polytope has an automorphism group - a set of symmetries in which the form is mapped onto itself while preserving the incidence of faces - and a figure is chiral if it cannot be mapped to its mirror image by rotations and translations alone.

Helices such as screws and propellers, and Mōbius strips, are chiral objects in three-dimensions.

Examples of chiral polytopes with maximum rotational symmetry are known in three, four and five dimensions. Devillers and Conder have now found examples in six, seven and eight dimensions; "We hope to be able to generalize," says Alice.

"It could be that eight is the biggest; it's hard to tell. It is a very big problem for a computer." The two are studying the output of computations to look for patterns, which could be extended.

"Computers lead us in the right direction and give examples, but never provide proof unless it is a finite question and you have all the examples."

"It's a bit like detective work, a combination of investigating examples to prove general results; you go back and forth between the two."

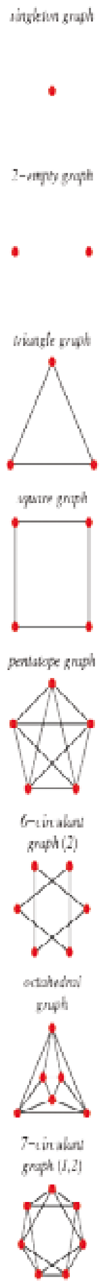
Permutation groups - where elements of a set are exchanged or permuted - can also be applied to other geometries. "I like geometries because I can draw them, they are concrete objects." In Perth, Devillers focuses on graphs, which are combinations of vertices and edges.

She is classifying locally s-distance transitive graphs. For these, the set of automorphisms fixing any vertex v has a single orbit on the vertices at distance 1 from v, on those at distance 2, and so on up to distance s.

Devillers liked maths from early in her schooling. "I did the national maths Olympiads every year from 13 to 18 in Belgium and went to one International Mathematical Olympiad. I do it because it's beautiful."

"It's the essence of science, the most precise. Either it's true or it's not. Some axioms are assumed, but that's always stated. The real work is always thinking. It's universal, you can explain it to any mathematician in the world, or any other world."

"And it's not just about proving theorems, but providing the best definition. Sometimes proofs are ugly, but another proof for the same theorem may be more elegant."



From turbulence to biofuel

Canadian Professor Andrew Pollard is an engineer who wears many hats. His 2009 visit was hosted by NZIMA and he spoke at the NZIMA programme workshop on energy, wind and water. He spoke to Jenny Rankine.

See also

- Pollard's webpage - http://me.queensu.ca/people/pollard
- C3.ca - www.c3.ca/ce/home_t.html
- The Sustainable Bioeconomy Centre – www.queensu.ca/sbc
- Queen's Collaborative Programme in Computational Science and Engineering - http://qcse.queensu.ca

Pollard occupies the Research Chair in Fluid Dynamics and Multi-scale Phenomena at Queen's University in Kingston, Ontario. He directs the university's Collaborative Programme in Computational Science and Engineering, and the Sustainable Bioeconomy Centre.

He has been the project leader for the High Performance Computing Virtual Laboratory, a cluster of powerful computers that now serves seven Ontario tertiary institutions, and president of the C3.ca Association, a national group of institutional users and providers of high performance computing.

He has been involved in computational and experimental fluid dynamics since the 1970s, focusing on turbulence, which he describes as the biggest unsolved classical problem in physics.

Earlier, he modelled combustion and radiation processes; now he simulates different flows, including airflow in the human windpipe, air bubbles in blood vessels and aerodynamics.

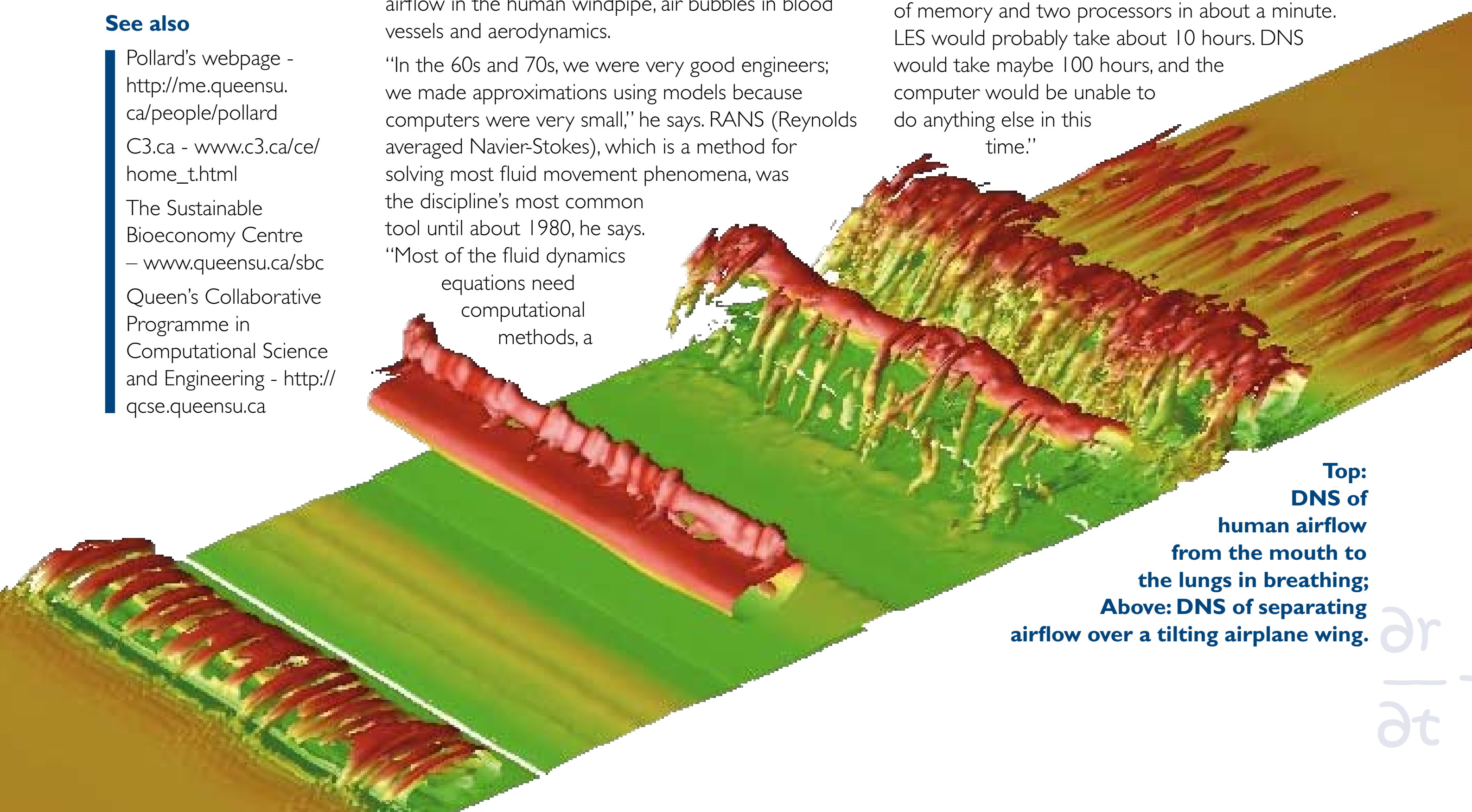
"In the 60s and 70s, we were very good engineers; we made approximations using models because computers were very small," he says. RANS (Reynolds averaged Navier-Stokes), which is a method for solving most fluid movement phenomena, was the discipline's most common tool until about 1980, he says. "Most of the fluid dynamics equations need computational methods, a combination of physics, chemistry, computer science and mathematics."

Large Eddy Simulation (LES) was first used in the early 1980s for engineering problems, although it had been used earlier for weather predictions. More recently, direct numerical simulation (DNS) solves fluid dynamics equations in three-dimensional space and time.

Pollard gives a building analogy: "From outside, you can see individual people going in and out; you can presume things about what happens inside. That's like RANS. LES is equivalent to seeing those people go in and out of lots of rooms."

"DNS is the equivalent of following a person in and out of those rooms and floors in three dimensions and real time in milliseconds."

"Let's assume the RANS algorithms about the building could be solved on a desktop computer with 1Gb of memory and two processors in about a minute. LES would probably take about 10 hours. DNS would take maybe 100 hours, and the computer would be unable to do anything else in this time."



Top: DNS of human airflow from the mouth to the lungs in breathing; Above: DNS of separating airflow over a tilting airplane wing.

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Pollard's DNS of airflow from the mouth to the lungs in breathing, left, uses 140 million points of measurement, developed from scans of many people.

He has been able to explain why asthma drugs clump at the back of the throat, and a Masters student found that the normal pipe flow of air from the trachea assumed by lung modellers is not what the airway delivers.

His team has also developed an algorithm that uses existing ultrasound systems to find, count and display air bubbles in heart operations. Queen's University is seeking industry interest in this discovery.

Pollard also simulates fluid dynamics in aerodynamics. "To compute the airflow over the whole body of a plane using DNS would take approximately one billion years," he says.

"It takes 64 processors three months to do a DNS of the airflow over a wing as it tilts 20 degrees, using 15 million measurement points."

Because DNS problems are so large, mathematicians isolate and partition critical regions; the wing simulation was divided into eight regions, solved simultaneously but in parallel.

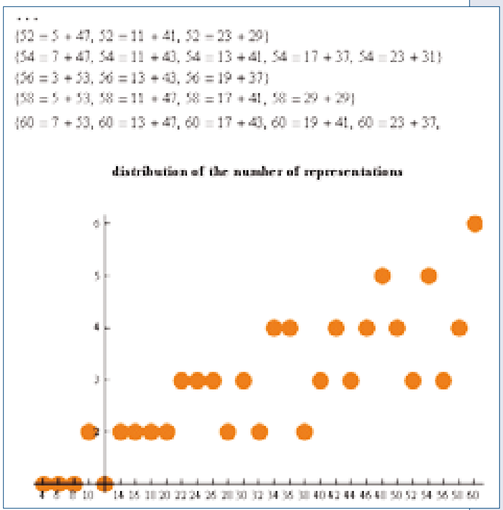
"Our hypothetical building may have 140 million rooms on 140 floors, so algorithms for each floor can be solved in parallel, because you only need to know the number of people on the stairs."

It's no wonder that Pollard became interested in high performance computing (HPC). The HPC Virtual Laboratory at Queen's is made up of more than 3,000 processors and terabytes of memory and disc space.

"The biggest ongoing costs are electricity and cooling, then support staff. In one USA installation they are building a separate power station just for the high performance computer."

Pollard says his "academic pathway has taken me in directions I hadn't anticipated". His most recent passion is the new Sustainable Bioeconomy Centre (SBC) at Queen's, which aims to help move economic reliance from 'old' oil to 'young' oil, or energy crops.

What hooked him into the issue was the question of how to transport fuel biomass in a pipeline. His team has applied for a patent for a transportable wood pellet, which doesn't have the storage and transport problems of existing pellets.



The number of ways an even number can be represented as the sum of two primes.

MATHEMATICAL EVENTS

6-10 July, University of New South Wales, Sydney
First Pacific Rim Mathematical Congress
www.primath.org/prima2009/

29 September - 2 October, Palmerston North
Pi in the Sky: Extending Mathematical Horizons, Biennial Conference of the NZ Association of Mathematics Teachers (NZAMT11)
www.nzamt.org.nz/nzamt11/

8-10 December, Albany, Auckland
Annual NZ Mathematics Colloquium, Albany Campus, Massey University

3-10 January 2010, Hanmer Springs
Groups, Representations and Number Theory, Annual NZMRI/NZIMA Summer Meeting
www.math.auckland.ac.nz/wiki/2010_NZMRI_Summer_Workshop

NOTABLE MATHS PROBLEMS

GOLDBACH CONJECTURE

That every even integer greater than 3 can be written as the sum of two primes

Also known as: The "strong", "even" or "binary" Goldbach conjecture because it implies the "weak", "odd" or "ternary" Goldbach conjecture that all odd numbers greater than seven are the sum of three odd primes. The conjecture does not specify that a number has to be the sum of only one pair of prime numbers.

Discipline: Number theory.

Originator: Prussian mathematician Christian Goldbach wrote to Leonhard Euler in 1742 proposing that every integer greater than two can be written as the sum of three primes. Euler replied that it follows that every even integer greater than two can be written as the sum of two primes. Euler's form has since been known by Goldbach's name.

Incentive: Proving one of the oldest unsolved problems in number theory and all mathematics. A $1million prize offered by publisher Faber & Faber for a proof submitted before April 2002 was never claimed.

Examples: 4 = 2+2, 6 = 3+3, 8 = 3+5, 10 = 5+5 = 3+7, 12 = 5+7, and so on.

Verified results: For smaller numbers, the strong Goldbach conjecture can be directly verified. One 1938 attempt laboriously verified up to $n \leq 10^5$, while a distributed computer search has verified the conjecture for $n \leq 10^{18}$.

State of play: There is little doubt among mathematicians that both conjectures are true; Euler replied to Goldbach: "That every even number is a sum of two primes, I consider an entirely certain theorem in spite of that I am not able to demonstrate it." No purported proofs are currently accepted by the mathematical community.

The weak Goldbach conjecture is fairly close to resolution, but the strong conjecture is much harder to verify. It has been shown that every even number $n \geq 4$ is the sum of at most six primes.

Statistical work on the probabilistic distribution of prime numbers presents informal evidence for the conjecture for sufficiently large integers.

Taming the monster

Computing aspects of a group whose size is a 54-digit number was the focus of Professor Robert Wilson's visit early in 2009. He talked with Jenny Rankine.

Affectionately called the Monster, this group has order 246 · 320 · 59 · 76 · 112 · 133 · 17 · 19 · 23 · 29 · 31 · 41 · 47 · 59 · 71, which reads 808017424794512875886459904961710757005754368000000000.

Using a computer, Wilson earlier identified two 196882 by 196882 matrices that together generate the Monster group.

However, performing computer calculations with these matrices is extremely expensive in time and storage – the matrices alone take up five gigabytes of disc space.

Wilson has been working with NZIMA Maclaurin Fellow Eamonn O'Brien at the University of Auckland since an earlier visit in 2002, and is also collaborating with Associate Professor Jianbei An.

They are developing more sophisticated ways of doing computations with the Monster: "We can study the effect of the matrices using computer programs," says Wilson.

Wilson and his former PhD student, Beth Holmes, found that if V is a 196882 dimensional vector space over the field with two elements, and H is a large subgroup of the Monster in which it is easy to perform calculations, then elements of the Monster can be stored as words in the elements of H and an extra generator T.

This makes it reasonably quick to calculate the action of one of these words on a vector in V.

Most finite simple groups belong to families, but there are 26 sporadic individual groups that don't belong to families and the Monster is the biggest. "They're telling us something very specific about symmetry, but we don't understand exactly what it is," says Wilson.

"The sheer size of the Monster is a challenge to figure out what it's there for. We've made lots of progress classifying its maximal subgroups."

During his visit Wilson was also finishing a textbook on finite simple groups aimed at graduate students. This built on his work as a co-author in 1985 of the landmark Atlas of Finite Groups: Maximal Subgroups and Ordinary Characters for Simple Groups.

Wilson's former supervisor John Conway and their three other co-authors' six-letter surnames make a matrix on the cover.

Wilson is based at Queen Mary, University of London. Before his five years there, he lived in Birmingham and still plays viola and violin in the Sinfonia of Birmingham, an amateur orchestra with a professional leader.

He believes there is some truth in the maxim about the link between maths and music, but also says that both disciplines demand obsession and hours of practice.

"At the end of secondary school when I was learning calculus there were hundreds and hundreds of exercises with the variations, very like practicing scales. I didn't find it boring because I could do most of the exercises; it was a challenge to pit your wits against the next problem and try to beat it."

See also

Aspects of the Atlas of Finite Groups are online at http://brauer.maths.qmul.ac.uk/Atlas/v3/

Order = 4030387200 = 210.33.52.73.17.

Awards and honours

John Butcher, an NZIMA Principal Investigator, was awarded an Honorary Fellowship of the European Society of Computational Methods in Sciences and Engineering, "for his outstanding contribution in the field of Computational Mathematics and Numerical Analysis" in September. His 75th birthday was also honoured at a conference in Greece.

Mike Hendy, another NZIMA PI, won a RSNZ New Zealand Science and Technology Medal, and the NZ Mathematical Society's annual Research Award late in 2008. They recognise Mike's seminal work on mathematical approaches to molecular ecology and evolution. His quantitative methodology forms an integral part of phylogenetic software packages.

Shaun Hendy, Director of our programme on Applications of Mathematics in the Nanosciences, has been appointed Deputy Director of the MacDiarmid Institute for Advanced Materials and Nanotechnology.

Professor **Peter Hunter**, an NZIMA PI, has won this year's World Class New Zealand Award in the Research, Science, Technology and Academia category. These awards, presented by KEA New Zealand and New Zealand Trade and Enterprise, celebrate some of New Zealand's tallest poppies. **Peter Hunter** was also appointed as the new chair of the Marsden Fund Council. He was a council member and convened its Mathematical and Information Sciences Panel from 2005 to 2008.

NZIMA co-director **Vaughan Jones** gave an invited lecture on the Poincare conjecture and the Riemann hypothesis at a symposium of Rutherford Medal winners in Dunedin in December. Vaughan was the first ever winner of the Rutherford Medal, in 1991. See www.odt.co.nz/on-campus/university-otago/34559/top-scientists-gather-dunedin

Andy Philpott, an NZIMA PI and Co-Director of our programme on Mathematical Models for Optimizing Transportation Services, is a member of the Norske Skog team that is a finalist in the 2009 Franz Edelman contest. This team was nominated for the role they have given Operations Research in achieving improved profitability.

Nic Smith, Director of an early programme on Modelling Cellular Function, has been appointed Professor of Computational Physiology at the University of Oxford, a remarkable achievement for someone under 40.