

From turbulence to biofuel



Canadian Professor Andrew Pollard is an engineer who wears many hats. His 2009 visit was hosted by NZIMA and he spoke at the NZIMA programme workshop on energy, wind and water. He spoke to Jenny Rankine.



Pollard occupies the Research Chair in Fluid Dynamics and Multi-scale Phenomena at Queen's University in Kingston, Ontario. He directs the university's Collaborative Programme in Computational Science and Engineering, and the Sustainable Bioeconomy Centre.

He has been the project leader for the High Performance Computing Virtual Laboratory, a cluster of powerful computers that now serves seven Ontario tertiary institutions, and president of the C3.ca Association, a national group of institutional users and providers of high performance computing. He has been involved in computational and experimental fluid dynamics since the 1970s, focusing on turbulence, which he describes as the biggest unsolved classical problem in physics.

Earlier, he modelled combustion and radiation processes; now he simulates different flows, including airflow in the human windpipe, air bubbles in blood vessels and aerodynamics.

"In the 60s and 70s, we were very good engineers; we made approximations using models because computers were very small," he says. RANS (Reynolds averaged Navier-Stokes), which is a method for solving most fluid movement phenomena, was the discipline's most common tool until about 1980, he says.

"Most of the fluid dynamics equations need computational methods, a

combination of physics, chemistry, computer science and mathematics."

Large Eddy Simulation (LES) was first used in the early 1980s for engineering problems, although it had been used earlier for weather predictions. More recently, direct numerical simulation (DNS) solves fluid dynamics equations in three-dimensional space and time.

Pollard gives a building analogy: "From outside, you can see individual people going in and out; you can presume things about what happens inside. That's like RANS. LES is equivalent to seeing those people go in and out of lots of rooms."

"DNS is the equivalent of following a person in and out of those rooms and floors in three dimensions and real time in milliseconds."

"Let's assume the RANS algorithms about the building could be solved on a desktop computer with 1 Gb of memory and two processors in about a minute. LES would probably take about 10 hours. DNS would take maybe 100 hours, and the computer would be unable to do anything else in this time."

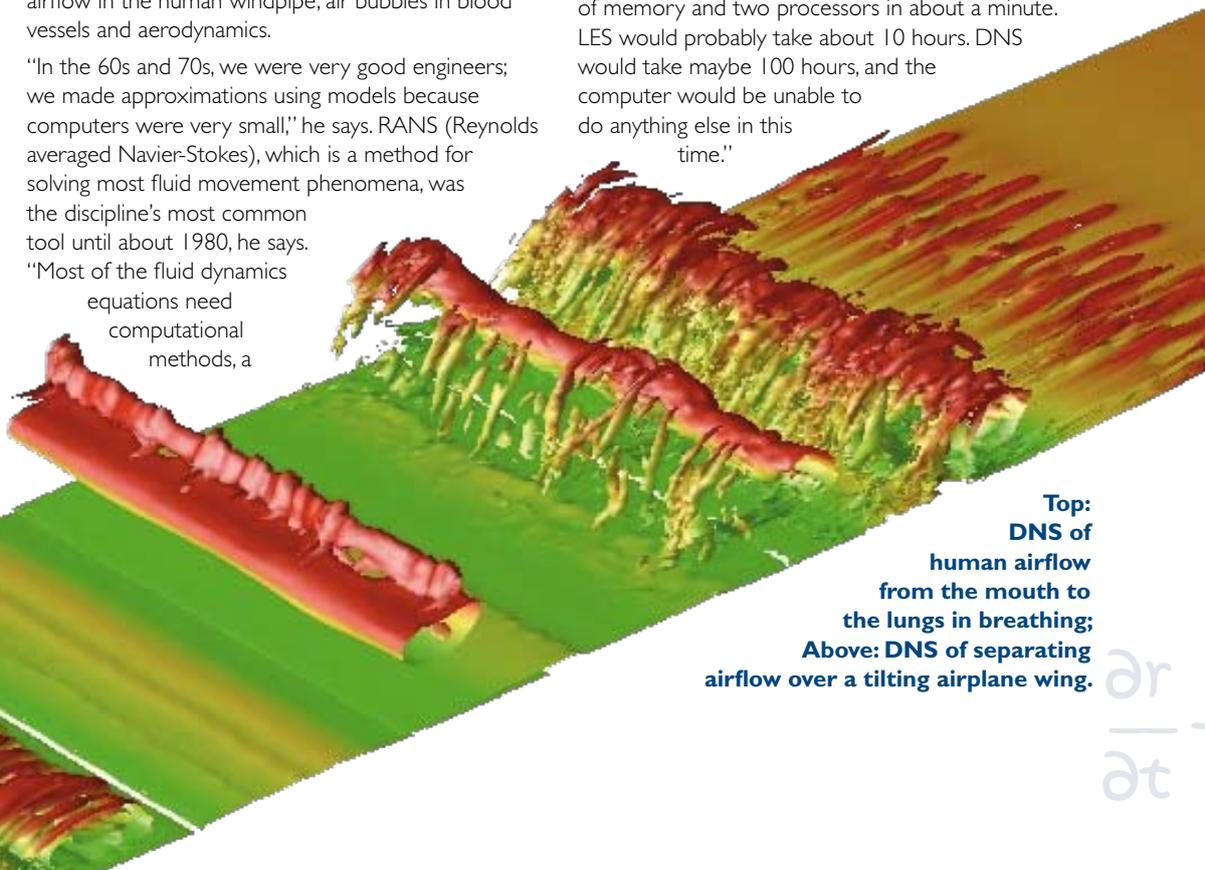
See also

Pollard's webpage - <http://me.queensu.ca/people/pollard>

C3.ca - www.c3.ca/ce/home_t.html

The Sustainable Bioeconomy Centre - www.queensu.ca/sbc

Queen's Collaborative Programme in Computational Science and Engineering - <http://qcse.queensu.ca>



Top: DNS of human airflow from the mouth to the lungs in breathing; Above: DNS of separating airflow over a tilting airplane wing.

$\frac{\partial r}{\partial t}$

Pollard's DNS of airflow from the mouth to the lungs in breathing, left, uses 140 million points of measurement, developed from scans of many people.

He has been able to explain why asthma drugs clump at the back of the throat, and a Masters student found that the normal pipe flow of air from the trachea assumed by lung modellers is not what the airway delivers.

His team has also developed an algorithm that uses existing ultrasound systems to find, count and display air bubbles in heart operations. Queen's University is seeking industry interest in this discovery.

Pollard also simulates fluid dynamics in aerodynamics. "To compute the airflow over the whole body of a plane using DNS would take approximately one billion years," he says.

"It takes 64 processors three months to do a DNS of the airflow over a wing as it tilts 20 degrees, using 15 million measurement points."

Because DNS problems are so large, mathematicians isolate and partition critical regions; the wing simulation was divided into eight regions, solved simultaneously but in parallel.

"Our hypothetical building may have 140 million rooms on 140 floors, so algorithms for each floor can be solved in parallel, because you only need to know the number of people on the stairs."

It's no wonder that Pollard became interested in high performance computing (HPC). The HPC Virtual Laboratory at Queen's is made up of more than 3,000 processors and terabytes of memory and disc space.

"The biggest ongoing costs are electricity and cooling, then support staff. In one USA installation they are building a separate power station just for the high performance computer."

Pollard says his "academic pathway has taken me in directions I hadn't anticipated". His most recent passion is the new Sustainable Bioeconomy Centre (SBC) at Queen's, which aims to help move economic reliance from 'old' oil to 'young' oil, or energy crops.

What hooked him into the issue was the question of how to transport fuel biomass in a pipeline. His team has applied for a patent for a transportable wood pellet, which doesn't have the storage and transport problems of existing pellets.

NOTABLE MATHS PROBLEMS

GOLDBACH CONJECTURE

That every even integer greater than 3 can be written as the sum of two primes

Also known as: The "strong", "even" or "binary" Goldbach conjecture because it implies the "weak", "odd" or "ternary" Goldbach conjecture that all odd numbers greater than seven are the sum of three odd primes. The conjecture does not specify that a number has to be the sum of only one pair of prime numbers.

Discipline: Number theory.

Originator: Prussian mathematician Christian Goldbach wrote to Leonhard Euler in 1742 proposing that every integer greater than two can be written as the sum of three primes. Euler replied that it follows that every even integer greater than two can be written as the sum of two primes. Euler's form has since been known by Goldbach's name.

Incentive: Proving one of the oldest unsolved problems in number theory and all mathematics. A \$1million prize offered by publisher Faber & Faber for a proof submitted before April 2002 was never claimed.

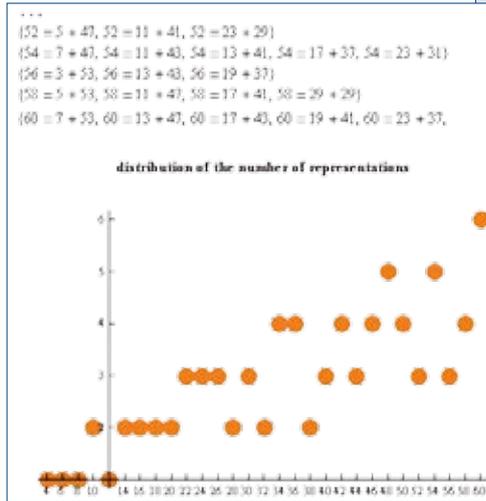
Examples: $4 = 2+2$, $6 = 3+3$, $8 = 3+5$, $10 = 5+5 = 3+7$, $12 = 5+7$, and so on.

Verified results: For smaller numbers, the strong Goldbach conjecture can be directly verified. One 1938 attempt laboriously verified up to $n \leq 10^5$, while a distributed computer search has verified the conjecture for $n \leq 10^{18}$.

State of play: There is little doubt among mathematicians that both conjectures are true; Euler replied to Goldbach: "That every even number is a sum of two primes, I consider an entirely certain theorem in spite of that I am not able to demonstrate it." No purported proofs are currently accepted by the mathematical community.

The weak Goldbach conjecture is fairly close to resolution, but the strong conjecture is much harder to verify. It has been shown that every even number $n \geq 4$ is the sum of at most six primes.

Statistical work on the probabilistic distribution of prime numbers presents informal evidence for the conjecture for sufficiently large integers.



The number of ways an even number can be represented as the sum of two primes.

MATHEMATICAL EVENTS

6-10 July, University of New South Wales, Sydney

First Pacific Rim Mathematical Congress

www.primath.org/prima2009/

29 September - 2 October, Palmerston North

Pi in the Sky: Extending Mathematical Horizons, Biennial Conference of the NZ Association of Mathematics Teachers (NZAMT II)
www.nzamt.org.nz/nzamtII/

8-10 December, Albany, Auckland
Annual NZ Mathematics Colloquium, Albany Campus, Massey University

3-10 January 2010, Hanmer Springs
Groups, Representations and Number Theory, Annual NZMRI/ NZIMA Summer Meeting
www.math.auckland.ac.nz/wiki/2010_NZMRI_Summer_Workshop

$$\frac{\partial u_i}{\partial x_j} = 0$$

Taming the monster

Computing aspects of a group whose size is a 54-digit number was the focus of Professor Robert Wilson's visit early in 2009. He talked with Jenny Rankine.

Affectionately called the Monster, this group has order $246 \cdot 320 \cdot 59 \cdot 76 \cdot 112 \cdot 133 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71$, which reads 80801742479451287588645990496171075700575436800000000.

Using a computer, Wilson earlier identified two 196882 by 196882 matrices that together generate the Monster group.

However, performing computer calculations with these matrices is extremely expensive in time and storage – the matrices alone take up five gigabytes of disc space.

Wilson has been working with NZIMA Maclaurin Fellow Eamonn O'Brien at the University of Auckland since an earlier visit in 2002, and is also collaborating with Associate Professor Jianbei An.

They are developing more sophisticated ways of doing computations with the Monster: "We can study the effect of the matrices using computer programs," says Wilson.

Wilson and his former PhD student, Beth Holmes, found that if V is a 196882 dimensional vector space over the field with two elements, and H is a large subgroup of the Monster in which it is easy to perform calculations, then elements of the Monster can be stored as words in the elements of H and an extra generator T .

This makes it reasonably quick to calculate the action of one of these words on a vector in V .

Most finite simple groups belong to families, but there are 26 sporadic individual groups that don't belong to families and the Monster is the biggest. "They're telling us something very specific about symmetry, but we don't understand exactly what it is," says Wilson. "The sheer size of the Monster is a challenge to figure out what it's there for. We've made lots of progress classifying its maximal subgroups."



During his visit Wilson was also finishing a textbook on finite simple groups aimed at graduate students. This built on his work as a co-author in 1985 of the landmark *Atlas of Finite Groups: Maximal Subgroups and Ordinary Characters for Simple Groups*.

Wilson's former supervisor John Conway and their three other co-authors' six-letter surnames make a matrix on the cover.

Wilson is based at Queen Mary, University of London. Before his five years there, he lived in Birmingham and still plays viola and violin in the Sinfonia of Birmingham, an amateur orchestra with a professional leader.

He believes there is some truth in the maxim about the link between maths and music, but also says that both disciplines demand obsession and hours of practice.

"At the end of secondary school when I was learning calculus there were hundreds and hundreds of exercises with the variations, very like practicing scales. I didn't find it boring because I could do most of the exercises; it was a challenge to pit your wits against the next problem and try to beat it."

See also

Aspects of the *Atlas of Finite Groups* are online at <http://brauer.maths.qmul.ac.uk/Atlas/v3/>

Awards and honours

John Butcher, an NZIMA Principal Investigator, was awarded an Honorary Fellowship of the European Society of Computational Methods in Sciences and Engineering, "for his outstanding contribution in the field of Computational Mathematics and Numerical Analysis" in September. His 75th birthday was also honoured at a conference in Greece.

Mike Hendy, another NZIMA PI, won a RSNZ New Zealand Science and Technology Medal, and the NZ Mathematical Society's annual Research Award late in 2008. They recognise Mike's seminal work on mathematical approaches to molecular ecology and evolution. His quantitative methodology forms an integral part of phylogenetic software packages.

Shaun Hendy, Director of our programme on Applications of Mathematics in the Nanosciences, has been appointed Deputy Director of the MacDiarmid Institute for Advanced Materials and Nanotechnology.

Professor **Peter Hunter**, an NZIMA PI, has won this year's World Class New Zealand Award in the Research, Science, Technology and Academia category. These awards, presented by KEA New Zealand and New Zealand Trade and Enterprise, celebrate some of New Zealand's tallest poppies.

Peter Hunter was also appointed as the new chair of the Marsden Fund Council. He was a council member and convened its Mathematical and Information Sciences Panel from 2005 to 2008.

NZIMA co-director **Vaughan Jones** gave an invited lecture on the Poincaré conjecture and the Riemann hypothesis at a symposium of Rutherford Medal winners in Dunedin in December. Vaughan was the first ever winner of the Rutherford Medal, in 1991. See www.odt.co.nz/on-campus/university-otago/34559/top-scientists-gather-dunedin

Andy Philpott, an NZIMA PI and Co-Director of our programme on Mathematical Models for Optimizing Transportation Services, is a member of the Norske Skog team that is a finalist in the 2009 Franz Edelman contest. This team was nominated for the role they have given Operations Research in achieving improved profitability.

Nic Smith, Director of an early programme on Modelling Cellular Function, has been appointed Professor of Computational Physiology at the University of Oxford, a remarkable achievement for someone under 40.

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