IMAges NZ

ISSUE FOUR

INSIDE

New Zealand Institute of Mathematics & its Applications

The patterns of tiny feet

Professor Ian Stewart, in New Zealand on a Seelye Fellowship late last year, tells the story of the bet that launched a new branch of science.

The Governor of California bet $25,000 in 1870 that a trotting horse is completely off the ground at some point.

"You can't tell with the naked eye," says Stewart, "but the governor's friend Edward Muybridge invented a camera that could freeze very fast motion and trotted a horse past a line of these cameras. And it was off the ground at one point." Muybridge went on to photograph the gaits of every large animal he could, down to dogs and cats, founding the branch of science called Gait Analysis.

From fossilised dinosaur footprints and skeleton structure, Gait Analysis has been able to analyse how these animals must have moved, and how quickly. And by analysing how people move, it is possible to spot problems before they become serious and deal with them. Stewart describes this as a fascinating application of the mathematics of rhythmic patterns.

One result of Stewart's work in the area was a prediction that the number of sinusoidal waves moving along a centipede would be either an integer or half an integer. Photographs bear out the prediction.

Stewart is a Professor of Mathematics at Warwick University in the UK. He started out as an abstract algebraist, and working on the dynamics of symmetry as a way to get closer to applied mathematics. "There is a lot of potential for dynamic systems modelling in biology. Biologists have realised that it's not enough to list the interactions of chemicals and genes, we need to look at how the whole system works - and they're maths questions."

Welcome

We hope you enjoy this fourth issue of IMAges, which contains a range of items about the work and interests of the New Zealand mathematical sciences community. There are profiles of two of the world's leading mathematical communicators, who were both in New Zealand last year, plus one of our most recent Maclaurin fellows.

Find out more from www.nzima.org

Marston Conder and Vaughan Jones
Co-Directors of the NZIMA

Stewart illustrates, with typical enthusiasm, the Fibonacci number sequence in nature; after two starting values, each number is the sum of the two preceding numbers.

"Networks of neurones create the rhythms underlying patterns of gait," says Stewart. "The vestibular system in our ears, three semi-circular canals, senses head movement and signals our neck and shoulder muscles to keep us balanced. We know the wiring diagram that sends those signals; it has an elegant mathematical structure. You can draw it on the surface of a cube, with canals at the centre of each of the six faces and eight muscle groups at the eight corners of the cube."

"Each canal sends inhibitory signals to the four nearest corners, and excitatory signals to the four furthest corners, so this symmetry group has 48 elements. When a neural system has a lot of symmetries, we can predict some of the dynamics because they are organised by the symmetries."

Stewart is more widely known outside the mathematical world for his science fiction writing, and his best-selling books about the science of science fiction novelist Terry Pratchett's creation, Discworld. Stewart's position at Warwick involves responding to media queries on mathematics-related issues, and producing popular lectures and maths items for media, including internet podcasts. He has won several awards for his active popularising of mathematics and related science areas.

"A challenge is that a lot of people think the maths they did at school represents the whole subject; that very few new discoveries are made. It doesn't mean maths is frozen if you don't hear about them."

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Stewart is enthusiastic about the public understanding of science, because "we need to get across why we're doing maths research, why it's interesting, and what they are getting out of it." One of the reasons for his visit was to share some ideas about building public understanding of maths.

"As soon as you start thinking of that, behind the scenes are massive amounts of maths, most of it quite new. Your MP3, CD and DVD players and GPS navigation, wouldn't work without error correcting code, for example."

"The Reed-Solomon code developed in the 1970s, based on very abstract algebra, is now the main error correcting system used in CD and DVD players. If you're driving and you go over a bump, this code makes sure the music comes out the way it should. The code can spot mistakes in the signal and transform it to what it should be. These devices work smoothly and seamlessly because of this underlying maths." *Jenny Rankine*

See also

Stewart's podcasts at www2.warwick.ac.uk/newsandevents/audio/more/symmetry/

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First NZMASP conference

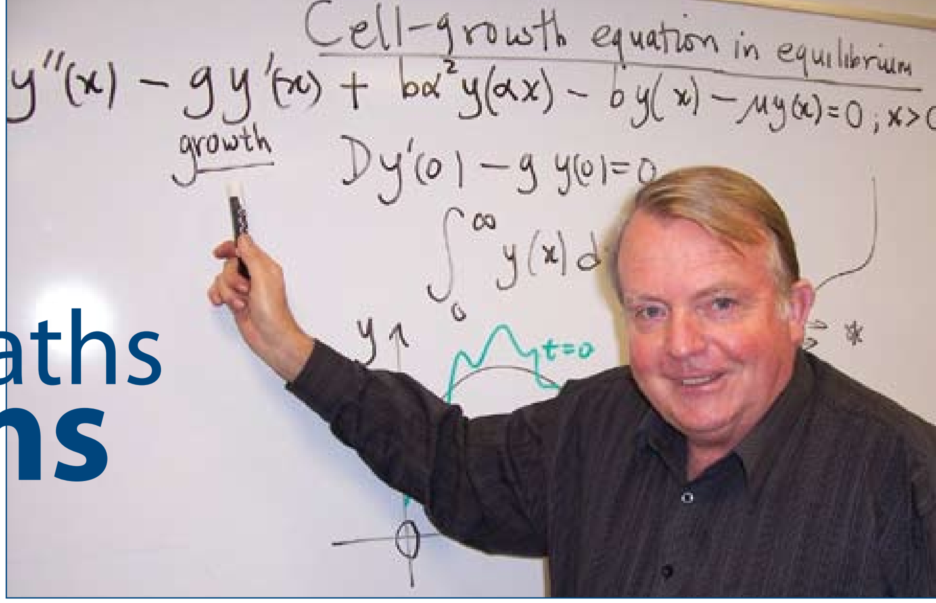
Nearly 40 postgraduate students from six New Zealand universities presented their research at the first New Zealand Mathematics and Statistics Postgraduate (NZMASP) Conference in Queenstown in November 2007. Topics varied from abstract algebra to physiological modelling, from Bayesian statistics to mathematics education, and all were well received.

Hannes Diener's talk, *The Dark Side of Constructive Reverse Mathematics*, won the NZIMA Best Presentation Award, while Michael Langton's presentation on *Surface Reconstruction with Piecewise Radial Basis Functions* earned him the Peoples' Choice Award sponsored by Hoare Research Software

Michael later revealed that searching for the algorithm to most clearly animate his talk led to the discovery that this algorithm gave the best selection of points for surface reconstruction.

Registration for the two-day event was free, due to sponsorship from the NZIMA and the University of Canterbury Mathematics and Statistics Department.

Dion O'Neale and Peter Humphries



Finding maths solutions

Professor Graeme Wake is surrounded by a ferment of mathematical problem-solving. Jenny Rankine explains.

He is Director of the Centre for Mathematics in Industry at Massey University in Albany. Every summer for the last four years, organisations have brought their maths-related problems to the Mathematics-in-Industry Study Groups of the Australian and New Zealand Industrial and Applied Mathematics (ANZIAM), the first time they had been held in New Zealand.

Some of the best applied maths minds in the country concentrated on solutions for TransPower, Fisher and Paykel, New Zealand Steel and other organisations in power supply, manufacturing, soil erosion, aeronautics, horticulture, tree growth, mining and financial markets. "Over three years we made substantial progress on 19 out of 20 problems," says Wake. "Many were completely solved in a week and some led to ongoing contracts."

"Financial maths is one of the fastest growing areas of applied maths - the futures and foreign exchange markets are heavily mathematical. Good maths models are crucial for survival in big markets. The New Zealand stock market is just starting to get involved, but the finance industry can't get the right graduates - we're not producing enough of them here."

Maths consultancy

Professor Wake also relishes the problems he works on as a private maths consultant. They range from large projects about controlling agricultural spray drift to smaller contracts. "The unique mix of academic and consulting mathematics is very enriching."

"I finished a problem this morning for a company producing materials to clean air conditioning systems in large buildings, to stop Legionnaires Disease and other bugs. They wanted an optimisation routine to reduce waste. The engineers knew what they needed but couldn't work out the relationship between the processes and the constraints. I managed to do the algorithm for Excel and the engineers wrote up the programme."

An even more local problem came from a family friend who manages a carpet warehouse. "She asked me about carpet offcuts; she didn't know how much was left in off-cut rolls and didn't want to waste staff time unrolling them. She knows how wide they are, so I gave her a solution and told her how to use it. All she had to input was the number of rings in the roll, the thickness of the carpet, the diameter of the hole in the middle and the outside of the roll, and that gave her the length. Industrial mathematicians need to be able to take problems that are not stated in mathematical terms, solve them and provide a user-friendly output."

"There is a lot of consulting work out there, although it's a challenge. There wouldn't be more than a couple of dozen people in the country earning their living by maths consultancy, although a lot of people do it on the side. Some of the most successful have interdisciplinary backgrounds in medical, agricultural or engineering fields and are able to talk the client's language. You learn it by doing; it's not so easy to train students in it - it's not like textbook maths."

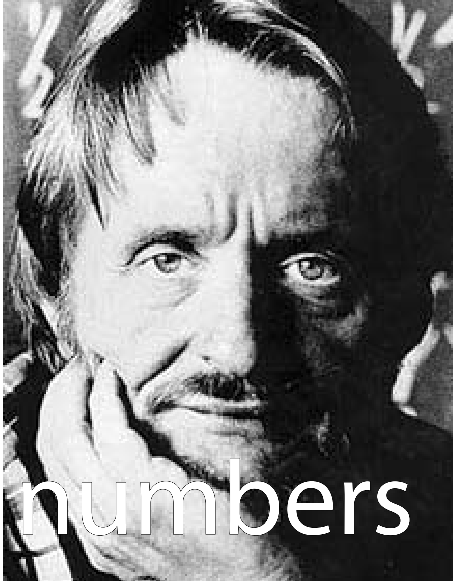
Wake is Professor of Industrial Mathematics at Massey and the NZIMA's full-time Maclaurin Fellow for 2007/08. The fellowship is named after Richard Maclaurin, the foundation Professor of Mathematics at Victoria University of Wellington more than 100 years ago, who later became a noted President of the Massachusetts Institute of Technology (MIT) in Boston.

During Wake's research year he is mounting a "full attack on non-local calculus", which is used in mathematical models of situations where cause and effect are separated by time, space or age. This interest grew from his earlier work on modelling cell growth, which is now used to quantify the growth of tumour cells and the effects of chemotherapy treatment.

"Calculus at year 13 is about rates of growth proportional to size now, but the rate of growth in cells is related to the size of the organism one division before. It's non-standard calculus - in fact, it's not in the text books. There are few procedures for the generic analysis of these types of problems."

Wake is investigating properties such as wavelengths and frequencies, and developing solution techniques for the functional differential equations involved. If he is successful, his work will relate to a wide range of applications.

Top: Graeme Wake. Below: A model of the effectiveness of shelter belts in stopping insecticide spray drift has resulted from a Maths-In-Industry Study Group, bottom.



Games and surreal numbers

Princeton University Professor John Conway, a NZIMA visiting Maclaurin Fellow, spoke to a packed Auckland crowd about the Game of Life during his second visit in January 2007.

He developed the game with a group of graduate students in the late 1960s, in an attempt to create a simple mathematical model of the birth, life and death of an organism with as few rules as possible. They tried triangular and hexagonal lattices, rules that include two and even three sexes and used poker chips, coins and shells with reams of paper before they found a viable balance between life and death.

In the game, players create an initial configuration of live (black) and dead (white) cells on a square grid and watch how shapes evolve and replicate themselves.

When it was publicised in 1970 in Scientific American, the Game of Life instantly developed a cult following. Conway offered a $50 prize for the first person to generate a configuration that would repeatedly produce gliders - moving shapes - and it was won in a month. In the following two decades the game has supposedly consumed more computer time world-wide than any other single activity. It made Conway famous, especially among computer buffs, and it also opened up a new field of maths research called cellular automata.

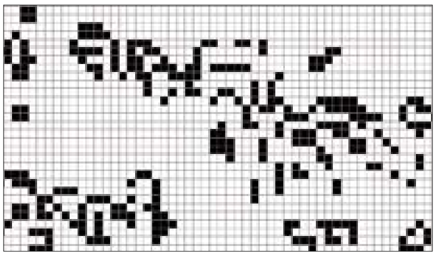
Conway's fascination with games and puzzles has led him to important discoveries in areas of mathematics from number theory to geometry. His first serious mathematical success, the discovery of what are now called surreal numbers, came to him when he was analysing the game of Go.

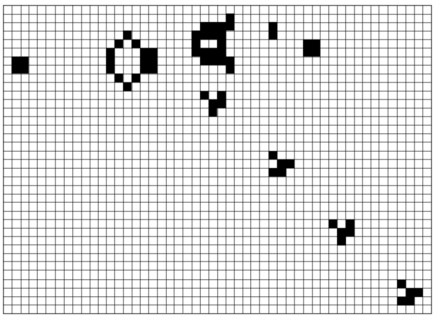
He noticed that the end of a game appeared like the sum of a lot of smaller games, and that certain games behaved like numbers. This made him think of algebraic fields of real and infinite numbers as well as numbers between 0 and 1. These surreal numbers are written as {L|R}: the left set contains a set of numbers below the surreal number and the right contains a set above the number.

Surreal numbers are Conway's favourite among his many maths discoveries: "I felt like I was seeing an enormous new world, richly covered in structure, which no one had ever seen before. It gives you a tingly feeling; I walked around in a daydream for six weeks."

The breakthrough was followed by an intense week of writing. "I started typing about 8am and finished at midnight every day for a week with an hour off in the middle, and at the end of it I had a book, On Numbers and Games. It lay around for a couple of years until I wrote a concluding chapter."

Jenny Rankine spoke with British mathematician John Conway.





Top: A version of the Game of Life implemented in JavaScript. See http://bendiken.net/2006/02/26/game-of-life-in-javascript

Below: American mathematician Bill Gosper won Conway's reward for creating infinite patterns with this "gun" - the combination of shapes at the top - that creates "gliders", patterns move diagonally downwards across the board.

He had felt guilty whenever he studied some children's game instead of doing more serious work, but after real mathematical success came from playing with games he decided the guilt should stop. "And as soon as I stopped feeling guilty, I had this annus mirabilis in 1980 - all sorts of things came out."

Conway has broken new ground in number theory, geometry and sphere packing and discovered one of the so-called sporadic simple groups, now called the Conway group.

During his second New Zealand visit, he co-wrote what he calls "a fun paper" on counting groups of a given order using gnus, moas and other "fascinating functions" with University of Auckland maths professor Eamonn O'Brien and PhD student Heiko Deitrich.

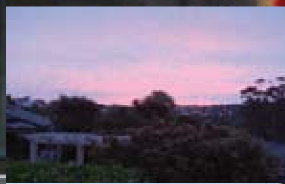
He has also collaborated with the NZIMA co-director Marston Conder on what Conway calls the Monster group. "It's an amazing object in 196,884-dimensional space, like a Christmas tree ornament with approximately 1020 vertices. It can't be broken into combinations of smaller symmetries. I'd love to know why it exists before I die - but I probably won't."

Another highlight of his visit was seeing Comet McNaught from Mt Eden. "I'm keenly interested in amateur astronomy - I've seen half a dozen comets in my life and this was the best."

A stroke in late 2006 has slowed Conway only slightly - he now writes with his left hand. He's distressed that he can't do some of the tricks he used to illustrate points in his lectures, such as balancing a coin on a bent coat hanger.

Conway hopes to return in summer of 2009, escaping again from the sleet and slush of an east coast USA winter. They'll need a bigger lecture theatre.

See the Game of Life at http://en.wikipedia.org/wiki/Conway%27s_Game_of_Life



Workshop for women

A two-day workshop for women researchers in the mathematical sciences organised by the NZIMA in November attracted 35 doctoral students and early career researchers from around the country.

The workshop aimed to help women enter and build research careers, and provide opportunities for networking. Sessions covered the PhD years, promotion, work-life balance, mentoring, and cultural diversity. Di McCarthy, Chief Executive of the Royal Society of New Zealand, also spoke on the status of women in New Zealand universities.

Additional sponsorship was provided by the Kate Edger Educational and Charitable Trust.

Teacher-student relationships

PhD student **Robin Averill** is exploring effective teacher-student relationships in Year 10 mathematics classrooms and their connection with mathematics learning. The project explores three ways in which teachers care for students: as individuals; for their mathematics learning; and as culturally located people.

Averill is aiming to discover key features of these aspects of caring, and the students' perceptions of their importance for their mathematics achievement. Participating in the study are six teachers and their classes, from three schools, made up of predominantly Maori, Pasifika and New Zealand European students.

It is thought that understanding how caring relationships are established and maintained is important to help improve students' access to mathematics learning during secondary school and in their future study. *Anna Meyer*

Minimum-energy distortions

When a material is physically changed from one form into another, such as by heating or stressing, it is expected that it will deform in such a way that a minimum of energy is spent.

Recently, important connections have been established between finding minimum-energy deformations, and problems in a branch of mathematics known as the calculus of variations.

The results of research in this area indicate that in certain cases, no minimum-energy deformation can be obtained, even though visually and intuitively it would be expected that this would be possible.

PhD student **Maarten Jordens** aims to show that minimum-energy deformations do not exist outside certain ranges, and that when they do, they are of a certain form. *Anna Meyer*

How the heart functions

PhD student **Vicky Wang** is studying how the left ventricle of the heart adapts its structure and function during cardiac disease. In diabetes or myocardial infarction, the heart cells adapt to physiological, geometric and loading changes in the heart muscle. This leads to thickening or thinning of the ventricular wall, and enhancement or degradation in regional muscle function.

Wang's project involves formulating mathematical models of left ventricle geometry and function, using clinical MRI data.

The new methodology and results from the project will provide an improved understanding of the underlying structural basis of ventricular mechanics in both normal and pathological conditions, and will reduce the number of human studies required to investigate cardiac disease and treatment in the future. *Anna Meyer*

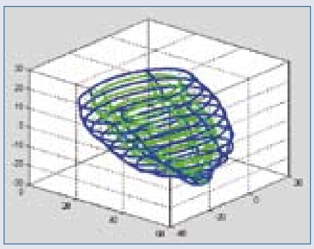
Modelling childbirth

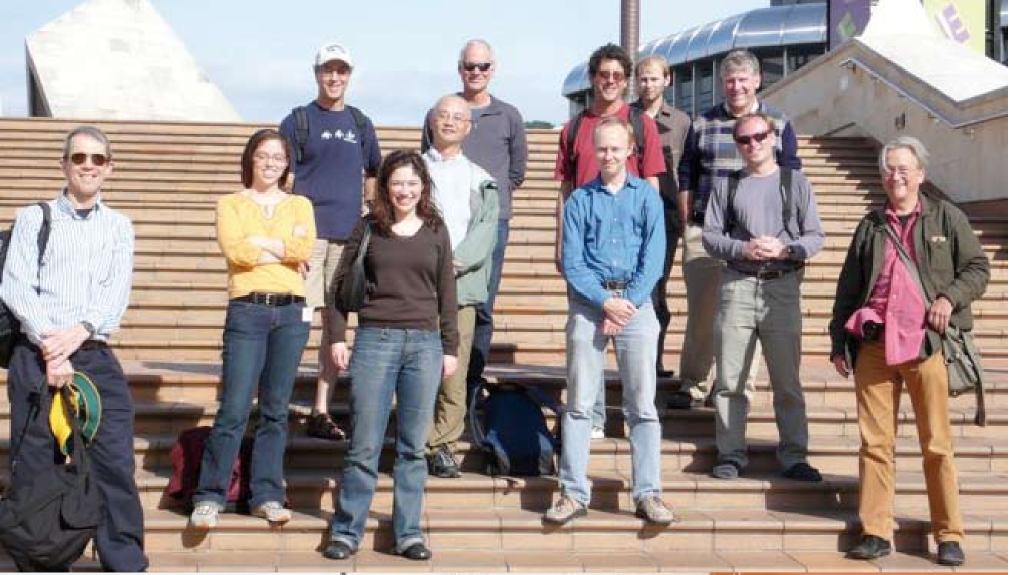
Research has suggested that female athletes involved in high-intensity sports for sustained periods have a higher probability of experiencing a prolonged second stage of labour during childbirth compared to non-athletes.

The mechanisms underpinning this complication are unclear, but may depend on the size or tone of the pelvic floor muscles. PhD student **Shannon Li** is studying the relationship between the size and tone of the pelvic floor muscles and the level of difficulty during childbirth. This involves generating anatomically realistic models of the pelvic floor for a female athlete and a non-athlete, as well as the fetal skull. So far, an initial modelling framework has been constructed to simulate the motion of the fetal head during delivery. *Anna Meyer*

> **"Is mathematics an act of creation or an act of discovery? Many mathematicians fluctuate between feeling they are being creative and a sense they are discovering absolute scientific truths."**
> *Marcus de Sautoy, Music of the Primes.*

Wang's model of deformation of the heart





Top: Dillon Mayhew and James Oxley convened the special session on Matroids, Graphs and Complexity; members at the City to Sea Bridge. Centre: Special session on Group Theory, Actions and Computation speakers, convened by Russell Blyth and Marston Conder. Below: Vaughan Jones, Rod Downey, Ruth Charney (AMS Vice-President) and Gaven Martin (NZMS President) enjoy the opening ceremony.

AMERICAN AND NEW ZEALAND MATHS SOCIETIES MEET

Almost 300 international and New Zealand participants enjoyed over 200 high quality presentations at the first Joint Meeting of the American and New Zealand Mathematical Societies, held in Wellington in December.

Plenary lectures were given by four speakers from the US and four from New Zealand – with all of the latter being principal investigators of the NZIMA (Marston Conder, Rod Downey, Gaven Martin and Matt Visser). In addition, 20 keynote lectures and 205 other talks were presented in 15 special sessions. A notable feature was the unexpected degree of connection between the talks, especially around the themes of computation and computability.

Special sessions covered a wide range of pure and applied mathematics as well as the history and philosophy of mathematics and mathematics education. These sessions enabled communities of researchers to interact, and start or continue collaborations. Significant progress was made on many problems and projects as a result of collaborations organised at the meeting.

Peter Humphries and Ratneesh Suri jointly won the Aitken Prize for best talk by a student; the inaugural New Zealand Mathematical Society (NZMS) Early Career Award went jointly to Noam Greenberg and Catherine McCartin; and the annual NZMS Research Award to Ernie Kalnins.

The meeting was supported by grants from the AMS and the NZMS; the NZIMA provided travel support worth a total of $6,000 to 14 graduate students as well as sponsoring one of the plenary speakers, Bruce Kleiner of Yale University.

Planing the crew

Oliver Weide is the latest in a procession of University of Auckland operations research students whose research has been implemented at Air New Zealand.

The airline first collaborated with Professor David Ryan, Weide's PhD supervisor and an NZIMA board member, in 1984. This led to the airline being the first to operate a computerised crew schedule.

Weide's research focused on integrating the airline's aircraft and crew schedules. Crew tours of duty and rosters were still worked out after aircraft routes are fixed, which limited the options.

"Flight schedules are cheaper if there are short turnarounds between flights. But when crew have to swap planes with tight turnarounds, a short delay in one flight can cause a chain of delays in many other flights. We developed a robustness measure for each connection in the previous Air New Zealand summer schedule, which penalised crew changing aircraft; the shorter the ground time, the higher the penalty.

"We integrated aircraft routing and crew tours of duty by going back and forth between the two problems iteratively. Each iteration increased the penalties for crew changing planes, making the solutions more robust, and resulting in much cheaper and more robust schedules."

The Optima Corporation has implemented the measure for Air New Zealand.

Jenny Rankine

NOTABLE MATHS PROBLEMS

TWIN PRIME CONJECTURE

There are infinitely many primes p such that p + 2 is also prime.

Simply: That the number of prime numbers that differ by two, such as 101 and 103, is infinite.

Discipline: Number theory.

Originator: Euclid, around 300BC.

Incentive: Being the first to solve a 2,300-year-old problem.

Partial proofs: In 1915, Norwegian mathematician Viggo Brun showed that the sum of reciprocals of the twin primes was convergent. This famous result was the first use of the Brun sieve and helped initiate the development of modern sieve theory, a set of techniques designed to estimate the size of sifted sets of integers.

From 1940, Paul Erdős showed that there is a constant $c < 1$ and infinitely many primes p such that $(p' - p) < (c \ln p)$ where p' denotes the next prime after p. This result was successively improved by Helmut Maier, Daniel Goldston and Cem Yildirim.

In 1966, Chinese mathematician Chen Jingrun used sieve theory to show that there are infinitely many primes p such that p + 2 is either a prime or the product of two primes, now known as Chen primes. Terence Tao and Ben Green built on this to show that there are infinitely many three-term arithmetic progressions of Chen primes.

Mathematicians believe the twin prime conjecture to be true, based on numerical evidence and the probabilistic distribution of primes.

Unusual aspect: Because it is easily understood by non-mathematicians, the twin prime conjecture is a popular target for pseudo-mathematicians who attempt to prove or disprove it, sometimes using only high-school mathematics.

NZIMA connection: Marcus du Sautoy, visiting Maclaurin Fellow in 2007, for whom the distribution of prime numbers is a major interest.

Awards and honours

ROD DOWNEY, one of the NZIMA principals and our first Maclaurin Fellow, has been elected a Fellow of the Association for Computing Machinery (FACM), for his contributions to computability and complexity theory, and he has also been awarded a James Cook Research Fellowship for 2008-2010.

ERNIE KALNINS, a key member of our new programme on Conformal Geometry, won the New Zealand Mathematical Society's Research Award for 2007, for his work on symmetries of PDEs, separable co-ordinates and superintegrable systems.

JOHN KERNOHAN, a member of the NZIMA Governing Board, won the 2007 Thomson Medal of the Royal Society of New Zealand, for outstanding and inspirational leadership in the management of science.

THE OPTIMA CORPORATION, a spin-off company created by NZIMA principal David Ryan and his colleagues, won one of four Technology Commendations from the Foundation for Research, Science and Technology in 2007, for the development of software systems for optimal use of resources by Andrew Mason and his team.

DAVID RYAN, a member of our Executive Committee and co-director of our programme on transportation modelling, has been elected to a Fellowship of the Institute for Operations Research and the Management Sciences (INFORMS).

MIKE SAUNDERS (Stanford), a member of our International Scientific Advisory Board, was elected an Honorary Fellow of the Royal Society of New Zealand in November 2007.

MATHEMATICAL EVENTS

16 - 20 June, Dunedin
Conference on Permutation Patterns, www.cs.otago.ac.nz/staffpriv/mike/PP2008/

7 - 8 July, Hamilton
New Zealand Statistical Association Conference 2008, http://nzsa.rsnz.org/NZSA2008/index.htm

14 - 18 July, Auckland
GLADE 2008 conference on numerical methods for differential equations and related problems, www.auckland-ode-2008.org/

21 - 25 July, Auckland
GLADE 2008 workshop on numerical methods for differential equations and related problems, www.auckland-ode-2008.org/

4 - 8 August, Massey, Albany
Research workshop on Parabolic Geometry and PDE, www.math.auckland.ac.nz/wiki/Research_Workshop_on_Parabolic_Geometry

8 - 12 December, Christchurch
7th Australia-New Zealand Mathematics Convention, www.math.canterbury.ac.nz/ANZMC2008/

15 - 19 December, Auckland
4th International Conference on Combinatorial Mathematics and Combinatorial Computing, http://www.cs.auckland.ac.nz/research/groups/theory/4ICC/index.html

"Because it is so useful, people think that mathematics only exists for its use, but actually that is not true. The development of maths is one of the rich streams of intellectual history." *Geoff Whittle, Wellington*