Mathematical poetry and tables

Dr Clemency Montelle, of Canterbury University, studies ancient mathematical texts in dead languages, combining her love of both fields.

"I took Greek, Latin and maths at uni because I enjoyed them at school," she said. "After four years, I came across a book on Euclid's elements in the original Greek and realised I could read it and understand the maths, and I was really interested in figuring out what mathematical texts in ancient languages meant."

Sanskrit was the next step. "Then I learnt classical Arabic and picked up Akkadian. Once you have a couple of languages under your belt you know the rubric for learning them," she says. Akkadian emerged in ancient Mesopotamia (modern Iraq) around 3,600 BCE* and was written on clay tablets. "The first of these were found only 150 years ago - it was the first mathematically literate culture."

Montelle says that the European pioneers of mathematical history neglected the arts of 'number crunching', such as the construction of function tables and algorithms, particularly in non-Western traditions such as the Sanskrit sciences of India between 900 and 1800CE**. There are very few historians working in this area, in or outside India.

She is part of three projects studying these ancient Sanskrit mathematical manuscripts. One is *Histoire des tables numériques,* a four-year international study led by Dominique Tournès of Réunion in the Indian Ocean. Montelle has also received a Marsden Grant to identify, catalogue and publish online all known Sanskrit table texts, with Kim Plofker, of Union College in New York. They will also critically edit three key unpublished texts of tables. Montelle also received a 2012 Rutherford Discovery Fellowship to study a wider selection of Sanskrit mathematical documents.

"There might be 25 different manuscript copies of one text. We want to collect as many copies as possible, translate and compare them and write a commentary," she says. Many technical words, such as hypotenuse, aren't in Sanskrit dictionaries, "so we have to determine them from the context".

Tables are so commonplace now that their significance is almost lost on modern mathematicians, she says. "They're the most valuable source for seeing how mathematicians actually computed things. We can read their theoretical ideas about the position of the sun, but the only place we see them calculating it is in tables." The pair will examine mostly astronomical tables, about eclipses, planetary motions, astrology and trigonometry.

Montelle "spent two summers at the Houghton Library of rare books at Harvard, cataloguing their 2,500 Sanskrit documents, because nobody could read Sanskrit." Many manuscripts are in India and also not catalogued, so she will be working with the Chennai Mathematical Institute and visiting libraries there.

"Indian mathematics was composed into verse that was meant to be memorised," says Montelle. "Quite technical mathematics and scientific ideas were fitted into memorable poetry. Long and complex numbers – say pi to ten decimal places - become more of a challenge when they have to be expressed in verse."

Rather than trying to find a geometrical demonstration of astronomical events, using careful proofs as in the European tradition, Indian mathematicians were more interested in the algorithms for computing the events, she says. "They had several ways to compute the position of the sun and tolerated multiple models, while the West was fixated on one unified solution."

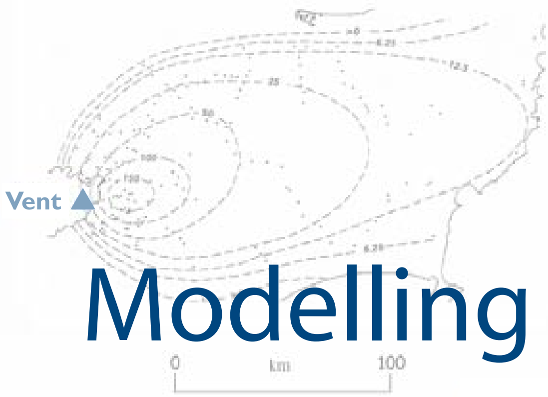
The projects will help explain how Indian astronomers developed new models, and how mathematical knowledge was transferred between the two traditions.

"As we explore complex areas such as quantum physics and coding theory for computers, science is becoming heavily computational and hearkens back to the emphases of Indian mathematicians," she says.

*BCE: Before the common era
*CE: Common era

Below: A copy of a manuscript in Sanskrit with a numerical table.

Inset: A close-up including the first 10 numbers. The place value base ten system of numeration – the use of 10 different glyphs to represent a number of any magnitude - first appeared in India. These are the beginnings of the numerals we use today.



$$c(x,t)=\frac{Q}{\sqrt{4\pi Dt}}e^{-\frac{(x-Ut)^2}{4Dt}}$$

Modelling **pollution** and **plumes**

Professor Robert McKibbin, of Massey University at Albany, models the transport of groundwater pollution and volcanic plumes in the atmosphere using very similar differential equations.

Waste from industry or agriculture commonly pollutes groundwater aquifers in most countries, says McKibbin. "The challenge is to find good ways of predicting where the pollutant will go."

One example of a pollutant was "sea water intrusion into fresh water aquifers near the coast in Queensland". There is a similar concern that drawing hot water out at Waiwera may be allowing sea water to seep into underground aquifers. However, his models are not based on one pollutant or geographical area, but are generic and meant to apply anywhere.

Aquifers are a kind of porous media, a structure containing many fine pathways, with a pressure difference across distance. "There has been a lot of work done on soluble pollutants over decades," he says, using standard computer packages for computational fluid dynamics. However, they use time-consuming numerical simulations.

"Because we can't see into the ground, and often can't drill cores, there are always unknown parameters in these models, and they have to be estimated."

"We're looking for more simplified models that are quicker and still accurate. They are semi-analytic models, using simpler differential equations to find simple solutions which can be written as formulae." PhD student Amjad Ali is modelling groundwater aquifers with different rock layers to improve the complexity of the model.

Their solutions were tested against experimental data from a site in Canada, where chloride ions, acting as a soluble tracer, were injected into a well and downstream measurements from the aquifer were monitored over a long period. "We found the data was a good fit with our model, which meant we could have told them what would happen next."

Modelling the flight of particles in the atmosphere is another generic problem using similar differential equations, which can be applied to volcanic ash, hydrothermal eruptions, dust, smoke and sand clouds, pollen, and horticultural spray drift. "Large particles fall fast, but tiny

ones tend to stay high in the air and blow a long way. The best local examples are smoke from Australian bushfires, where the dropping speed is almost negligible, and volcanic plumes from the central North Island. Organisations managing natural disasters need to know where that ash is going to fall."

Their model predicted the same shape as measured data from historic Taupo eruptions, where the thickness of drill core layers has been plotted on maps. "Volcanologists are keen to know what's going on in volcanic plumes, so we're now trying to predict the release height of particles in the plume."

PhD student Sharleen Harper, now working for NIWA, used the same techniques to calculate the way that spray droplets spread after being released in an orchard.

"There's enormous commercial investment in computer programs used to calculate movement of groundwater pollution," says McKibbin. "They use the same the differential equations we do. They're not wrong, but we're investigating less costly ways to solve those equations. The data from underground is pretty scarce anyway. Complicated models are not particularly useful when you have only a few bits of data."

Awards and honours

Roy Kerr (University of Canterbury); the 2013 Einstein Medal for his 1963 solution to Einstein's gravitational field equations.

Rod Downey (Victoria University of Wellington); 2011 Hector Medal from the Royal Society of NZ.

Robert McKibbin (Massey University); 2012 ANZIAM Medal (Australian and NZ Industrial and Applied Mathematics).

Rob Goldblatt (VUW); 2012 Jones Medal from the RSNZ.

James Russell (University of Auckland); 2012 NZ Prime Minister's Prize for Young and Emerging Scientists.

Adam Day (VUW); 2011 Sacks Prize for the best PhD thesis in logic and 2011 Hatherton Award from the RSNZ.

James Sneyd (UA); 2014 Maclaurin Lectureship by the American and NZ Mathematical Societies.

Clemency Montelle (UC); a five-year Rutherford Discovery Fellowship.

Noam Greenberg (VUW); a five-year John Templeton Turing Research Fellowship.

Marston Conder (UA), **Rod Downey** (VUW), **Vaughan Jones** (UA and Vanderbilt University) and **Gaven Martin** (Massey University); inaugural Fellows of the American Mathematical Society.

A special mention for pure and applied mathematics, which gained the highest average quality score (5.81) of all subjects in the latest Performance Based Research Fund evaluation, with 31.50 As (26.5%) and 50.23 Bs (42.2%). Congratulations to all who contributed to this outstanding achievement!

Top: A model of the thickness of deposits of volcanic fragments - tephra - from a Taupo eruption.

08

The orientation in space of the principal stresses in the San Fernando Valley in California.

Fisheries and tectonic stresses

Dr Richard Arnold, of Victoria University, is using statistical methods to estimate the size of New Zealand fisheries, and tectonic stresses in earthquake zones.

No one knows the total harvest, combining commercial, recreational, customary and illegal fishing, from any fishery area or stock in New Zealand, says Arnold.

In 2011, Ngāti Kahungunu wanted to know how much was being taken from the waters in their region, and whether their people were using recreational or customary licences to fish. Arnold and Masters student Kylie Maxwell, with the help of the iwi and the Ministry of Fisheries, collated the total harvest of 10 key fish species in the iwi area between 2007 and 2010.

Maxwell obtained commercial catch data from the ministry, and the results of recreational fishing surveys at local boat ramps. She obtained permission to study the records of 41 Māori fisheries officers, who issue customary permits for the coast between Wairoa and Cape Palliser. She also counted illegal fishing catches from the Ministry of Fisheries' conviction database.

Results showed that Ngāti Kahungunu people were largely using recreational fishing rights to fish for rock lobster, pāua and kina. Customary licences were often used for special events "when they might want a lot of pāua at once", says Arnold. Maxwell's estimate of total catch and species were illustrated in heat maps of the ocean showing the size of the catch across the Ngāti Kahungunu area, and pointed to a need for more consistent and higher quality data, especially on the recreational catch.

Antarctic toothfish

Arnold worked with a PhD student, Darcy Webber, to build a statistical model of the total Antarctic toothfish population, using commercial catch data. This fish, which has only been caught commercially since 1997, lives for at least 50 years, growing up underneath inshore sea ice and migrating further from shore later in life. "If you don't take that migration into account when you estimate the population" says Arnold, "you can seriously overestimate the size of it."

Webber integrated commercial catch data with scientific trawling surveys where fish are tagged, released and then later recaptured by the commercial fishery. The standard multi-stage statistical regression models work backwards from landed catch to how many fish there are in the sea, based on many assumptions. "We needed to examine all the assumptions in these models, including how fish grow, how well-mixed and distributed fish populations are in the sea, and how efficient fishing gear and fishing techniques are in capturing the fish."

The study is not finished, but Webber's initial estimate is that the toothfish population has dropped by approximately 15 percent since commercial fishing began, in contrast to some perceptions that commercial fishing has devastated the species.

Tectonic stress

In an earthquake the tectonic stresses in the earth overwhelm a planar weakness and two blocks of rock slide against each other. Arnold, with collaborator John Townend, has developed new methods for estimating the likely orientations of these earthquake fault planes.

"The statistical problem is that you get a very fuzzy view of each earthquake – the signal observed at seismometer stations reveals the geometrical properties of the earthquake, but it is wrapped in seismological noise added as the wave travels. Estimating the fault orientation and direction of the slip along the fault is difficult - you have to separate the signal from the noise."

"Solving this problem needed a new branch of directional statistics, of geometrical objects oriented in space that can have the multiple reflectional symmetries that earthquake fault planes do. My work with collaborator Peter Jupp has introduced new statistical methods that allow us, for example, to test whether the dominant stress is horizontal."

Arnold used the new methods to test whether the recent major earthquakes in Christchurch had changed the pattern of tectonic stress around the city. "Between the September 2010 and February 2011 major earthquakes the average location of the smaller earthquakes moved steadily eastward, but we found that there hasn't been any change in the orientation of the underground stresses driving those earthquakes."

$$\tan(2\emptyset_G^m) = \frac{2[R_{11}R_{21} + vR_{12}R_{22}]}{(R_{11}^2 - R_{21}^2) + v(R_{12}^2 - R_{22}^2)}$$