IMAges NZ

ISSUE THIRTEEN

ABI Biomechanics for breast cancer imaging team: From back left - Duane Malcolm, Prasad Babarenda Gamage, Barbara Curteis; front - Sally Malcolm, Poul Nielsen, Martyn Nash, Gill Nash, Jessica Jor

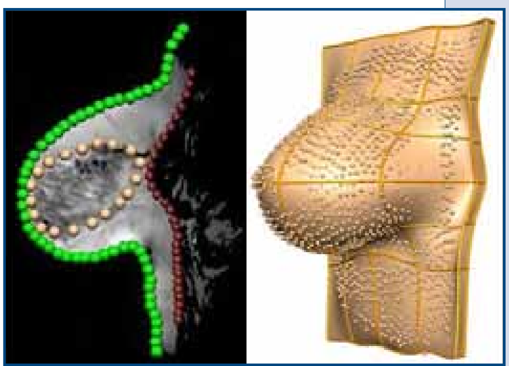
Modelling human bodies to detect disease

Professor Martyn Nash, Associate Director of Research at the University of Auckland Bioengineering Institute (ABI) and Deputy Head of the Department of Engineering Science, is working with teams of mathematicians to build models of the human breast and the beating human heart to help detect and prevent cancer and heart failure.

"There is a wealth of information about the breast from medical imaging," says Nash; "magnetic resonance imaging (MRI), computer topography (CT) scans, x-ray mammography, and ultrasound, as well as new modalities like breast tomosynthesis and thermal imaging, which haven't been clinically validated yet. We are developing integrated mathematical and computational models to bring all of that information together." He co-leads this project with Professor Poul Nielsen.

The radiologists and surgeons who they work with want accurate ways of finding suspicious lumps, micro-calcifications and new localised blood vessels, which can be early indicators of cancer." An MRI might show new blood vessels forming in a specific part of the breast, but they don't show up on mammograms; if we can co-locate that with micro-calcifications, which are generally visible in mammograms but not MRI, it provides extra diagnostic information," says Nash.

Hospital staff also want to be able to tell more accurately whether tiny suspicious points in an image taken when a woman is lying face down are the same as those found in a standing mammogram when her breast is compressed at two different angles.



A personalised biomechanical computer model of a woman's breast.

"It can be really hard to find the same feature in an ultrasound image that you see in a mammogram, and sonographers can sometimes spend half an hour looking for it. We want to help make that quicker and more reliable. Our mathematical modelling tools can also provide a way to validate new imaging technologies."

"We apply the laws of physics to soft tissue using calculus - derivatives and integrals. We use ▶2

Welcome

This is a special issue of IMAges to mark New Zealand's involvement in the Mathematics of Planet Earth initiative (MPE2013, mpe2013.org). Mathematics and statistics play a key role in the study of processes in the earth's mantle, oceans and atmosphere, as well as the complex systems supporting life and human society (including economic and financial systems, resource management, health care and communications).

We hope you enjoy reading the articles in this issue that highlight a range of contributions being made here in New Zealand.

We are grateful to the University of Auckland (Bioengineering, Mathematics and Statistics), Massey University (IMS Albany), Victoria University of Wellington (MSOR) and the University of Canterbury (Maths & Statistics) for their sponsorship of this issue.

Marston Conder and James Sneyd, Editors

INSIDE

Mathematics of Planet Earth 2013

taurangi

◀ I triangulating geometry to compare and we use statistics to check a model's validity and accuracy in clinical trials." The team uses images from patients at Auckland Hospital to create a three-dimensional computer model of each individual woman's breast.

Clinical software tools written by Dr Duane Malcolm are about to be delivered to the hospital. "We will provide a 3-D view and enable doctors to click a point on one 2-D image and see where it appears on another image," says Nash. "We have to solve highly non-linear equations to extract transformations that will allow them to point and click in real time."

The models need to be accurate to within 5mm. "We can get to within 10-15mm now, without taking different types of tissue into account," says Nash. PhD student Prasad Babarenda Gamage is estimating the mechanical properties of different tissue types to make the model more accurate.

Nash is using similar mathematics on another new project to model heart failure. "To look at the passive elastic properties of the heart we use the same partial differential equations we use for the breast and any other soft organs. For the active contraction of heart muscle we use ordinary differential equations." Nash has been working in this area since his PhD 20 years ago.

"Imaging gives us loads of information – you can get many different types of image data from an MRI scanner, more than our maths techniques can handle." The project will use motion imaging from MRI tagging; cine-anatomical imaging; diffusion tensor imaging; and histological images from the heart to validate the techniques.

Nash is working with Associate Professors Alistair Young and Ian LeGrice, and Dr Vicky Wang who will model abnormal heart function during heart failure. "Hearts get bigger, their fibre structure changes, and the ability of the heart muscle to contract is compromised by changes to sub-cellular processes such as calcium cycling and signal transduction."

"In systolic heart failure, the heart loses its ability to contract; in diastolic heart failure it has trouble relaxing and filling with blood", says Nash. There are good drug therapies for systolic but not for diastolic failure, which is sometimes difficult to diagnose and still poorly understood.

"We want to use mathematical models to merge the different types of imaging of individual people's hearts to categorise them more accurately and earlier, and to find out why their hearts start to fail."

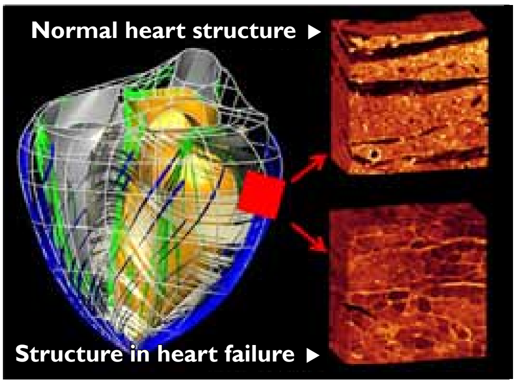
See also

www.abi.auckland.ac.nz/uoa/home/about/our-research/projects/biomechanics-for-breast-imaging

www.abi.auckland.ac.nz/uoa/home/about/our-research/projects/cardiac-mechanics

"Many mathematicians have a natural affinity with music. Mathematics departments invariably have little trouble assembling an orchestra from the ranks of their members."

Marcus de Sautoy, *Music of the Primes*



$$\frac{\partial \sigma^{ij}}{\partial x_i} + pb^j = pa^j$$

MATHEMATICAL EVENTS

June 24-28, 2013, Shanghai, China
Second Pacific Rim Mathematical Association (PRIMA) Congress, meeting.healife.com/prima2013/en/index.asp

October 1-4, Wellington
Absolutely Positively Mathematics & Statistics (NZ Assn of Maths Teachers Conference), nzamt13.org.nz

November 24-27, Hamilton
Joint Conference of the NZ Statistical Association and the Operations Research Society of NZ, smiller@stats.waikato.ac.nz or vanessa.cave@agresearch.co.nz

November 24-29, Kiama, NSW, Australia
Lighthouse DELTA 2013 (Conference on the teaching and learning of undergraduate mathematics and statistics), www.delta2013.net

December 2-4, Wellington
4th Wellington Workshop in Probability Theory and Mathematical Statistics, msor.victoria.ac.nz/Events/WWPMS2013

December 3-5, Tauranga
NZ Mathematical Society Colloquium nzmathsoc.org.nz/colloquium2013/home.php

January 13-17, 2014, Te Anau
NZMRI summer meeting (on Operator Algebras), www.maths.otago.ac.nz/nzmri14

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$$D_{n,n'} = \sup_x |F_{1,n}(x) - F_{2,n'}(x)|,$$

The statistics of wellbeing

Canterbury University PhD student Lisa Henley wants to picture the links between countries where people flourish and the values which enable them to do that.

"Gross domestic product (GDP) was developed in the USA to help production planning during World War II," says Henley, "and was never intended as a measure of national welfare. But GDP misses crucial parts of a country's wellbeing – those outside of monetary exchange."

Henley used free online global datasets about human wellbeing, flourishing and life satisfaction, which include up to 300 variables. She used a genetic algorithm and linear regression to select the most descriptive.

"These algorithms model natural selection and are a great way to find the best solution to a problem," says Henley. "You start with a population that represents your problem – various subsets of the 300 variables - constructed as binary elements."

"A binary tournament compares one set of variables to another and the fittest one progresses to the next level. All the winners are then resampled, swapping a length of their chromosome. The new chromosomes are tested for fitness, recombined with their parents, and winners proceed to the next level."

The algorithm repeats itself, calculating the average fitness of the result after each iteration. "Once it's been stable for 20 runs, it stops. I ended up with just over 100 variables, which is still a huge amount. The average years of secondary schooling for girls is coming through as really important."

The next step is to use genetic and clustering algorithms to find groups of countries according to levels of flourishing. "I'm using a multi-component fitness function, including some fairly cool non-parametric statistics to test the differences between the cluster populations."

Henley wants to represent the results visually, and may develop something similar to Gapminder software, which converts up to three variables over time into animated and interactive graphics. "Unfortunately, data available around inequality is still minimal."

When her data is graphed, she will look for "a happy people, happy planet 'sweet spot', and associate these with the values of countries. She will use data from the World Values survey to find "an ideal combination of values that lead to a flourishing life on a flourishing planet".

Henley says that much of the research into human flourishing "appears to be top-down, with variables being pre-defined. But statistics offers a wealth of methodologies for letting the data express the essence of a concept."

Henley attributes her interest in human flourishing to her time working in London as a research and development manager for Loyalty, the Sainsbury supermarket chain reward card, during the global bank crisis. There was a lot of discussion about whether buying more consumer goods was all there was to happiness, she says. "We think that increasing our wealth will increase our wellbeing, but the research shows that once we can meet our basic needs, there is a threshold after which further wealth doesn't increase our life satisfaction substantially."

See also

www.gapminder.org/
www.happyplanetindex.org/data/
www.prosperity.com/
www.tableausoftware.com/products/public

Notable maths problems

ERDÖS–FABER–LOVÁSZ CONJECTURE

If a graph (network) is made up of k complete subgraphs, each with k vertices, and having at most one vertex in common between any two of them, then the vertices of the graph can be coloured with k colours, such that adjacent vertices have different colours.

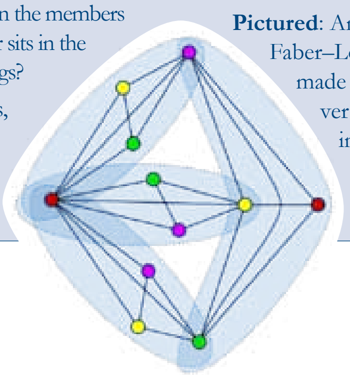
Simply: Suppose that an organisation has k committees (corresponding to complete subgraphs), each consisting of k members (corresponding to graph vertices), with any two committees having at most one member in common. All committees meet in the same room, which has k chairs (corresponding to vertex colours). Is it possible to assign the members to chairs so that each member sits in the same chair for all their meetings?

Discipline: Combinatorics, discrete mathematics

Originators: Hungarian mathematicians Paul Erdős and László Lovász and American mathematician Vance Faber formulated this innocent-sounding conjecture at a party in 1972, thinking it was trivial.

Incentive: When the authors realised the conjecture was harder than expected, Erdős offered US$50 for a proof that it was true, later raising this to US$500.

Progress: In 1988, W.I. Chiang and Eugene Lawler proved that the edge-colouring number of the graphs in the conjecture is at most $3k/2 - 2$, and four years later Jeff Kahn improved this to $k + o(k)$. In 2008, Abdón Sánchez-Arroyo established the truth of an equivalent version of the conjecture for dense hypergraphs.



Pictured: An instance of the Erdős–Faber–Lovász conjecture: A graph made up of four cliques of four vertices each, any two of which intersect in a single vertex, can be four-coloured.