IMAges NZ

ISSUE THIRTEEN

ABI Biomechanics for breast cancer imaging team: From back left - Duane Malcolm, Prasad Babarenda Gamage, Barbara Curteis; front - Sally Malcolm, Poul Nielsen, Martyn Nash, Gill Nash, Jessica Jor

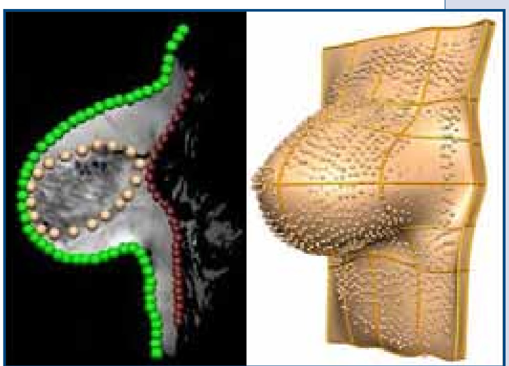
Modelling human bodies to detect disease

Professor Martyn Nash, Associate Director of Research at the University of Auckland Bioengineering Institute (ABI) and Deputy Head of the Department of Engineering Science, is working with teams of mathematicians to build models of the human breast and the beating human heart to help detect and prevent cancer and heart failure.

"There is a wealth of information about the breast from medical imaging," says Nash; "magnetic resonance imaging (MRI), computer topography (CT) scans, x-ray mammography, and ultrasound, as well as new modalities like breast tomosynthesis and thermal imaging, which haven't been clinically validated yet. We are developing integrated mathematical and computational models to bring all of that information together." He co-leads this project with Professor Poul Nielsen.

The radiologists and surgeons who they work with want accurate ways of finding suspicious lumps, micro-calcifications and new localised blood vessels, which can be early indicators of cancer." An MRI might show new blood vessels forming in a specific part of the breast, but they don't show up on mammograms; if we can co-locate that with micro-calcifications, which are generally visible in mammograms but not MRI, it provides extra diagnostic information," says Nash.

Hospital staff also want to be able to tell more accurately whether tiny suspicious points in an image taken when a woman is lying face down are the same as those found in a standing mammogram when her breast is compressed at two different angles.



A personalised biomechanical computer model of a woman's breast.

"It can be really hard to find the same feature in an ultrasound image that you see in a mammogram, and sonographers can sometimes spend half an hour looking for it. We want to help make that quicker and more reliable. Our mathematical modelling tools can also provide a way to validate new imaging technologies."

"We apply the laws of physics to soft tissue using calculus - derivatives and integrals. We use ▶2

Welcome

This is a special issue of IMAges to mark New Zealand's involvement in the Mathematics of Planet Earth initiative (MPE2013, mpe2013.org). Mathematics and statistics play a key role in the study of processes in the earth's mantle, oceans and atmosphere, as well as the complex systems supporting life and human society (including economic and financial systems, resource management, health care and communications).

We hope you enjoy reading the articles in this issue that highlight a range of contributions being made here in New Zealand.

We are grateful to the University of Auckland (Bioengineering, Mathematics and Statistics), Massey University (IMS Albany), Victoria University of Wellington (MSOR) and the University of Canterbury (Maths & Statistics) for their sponsorship of this issue.

Marston Conder and James Sneyd, Editors

INSIDE

Mathematics of Planet Earth 2013

tauranga

◀ I triangulating geometry to compare and we use statistics to check a model's validity and accuracy in clinical trials." The team uses images from patients at Auckland Hospital to create a three-dimensional computer model of each individual woman's breast.

Clinical software tools written by Dr Duane Malcolm are about to be delivered to the hospital. "We will provide a 3-D view and enable doctors to click a point on one 2-D image and see where it appears on another image," says Nash. "We have to solve highly non-linear equations to extract transformations that will allow them to point and click in real time."

The models need to be accurate to within 5mm. "We can get to within 10-15mm now, without taking different types of tissue into account," says Nash. PhD student Prasad Babarenda Gamage is estimating the mechanical properties of different tissue types to make the model more accurate.

Nash is using similar mathematics on another new project to model heart failure. "To look at the passive elastic properties of the heart we use the same partial differential equations we use for the breast and any other soft organs. For the active contraction of heart muscle we use ordinary differential equations." Nash has been working in this area since his PhD 20 years ago.

"Imaging gives us loads of information – you can get many different types of image data from an MRI scanner, more than our maths techniques can handle." The project will use motion imaging from MRI tagging; cine-anatomical imaging; diffusion tensor imaging; and histological images from the heart to validate the techniques.

Nash is working with Associate Professors Alistair Young and Ian LeGrice, and Dr Vicky Wang who will model abnormal heart function during heart failure. "Hearts get bigger, their fibre structure changes, and the ability of the heart muscle to contract is compromised by changes to sub-cellular processes such as calcium cycling and signal transduction."

"In systolic heart failure, the heart loses its ability to contract; in diastolic heart failure it has trouble relaxing and filling with blood", says Nash. There are good drug therapies for systolic but not for diastolic failure, which is sometimes difficult to diagnose and still poorly understood.

"We want to use mathematical models to merge the different types of imaging of individual people's hearts to categorise them more accurately and earlier, and to find out why their hearts start to fail."

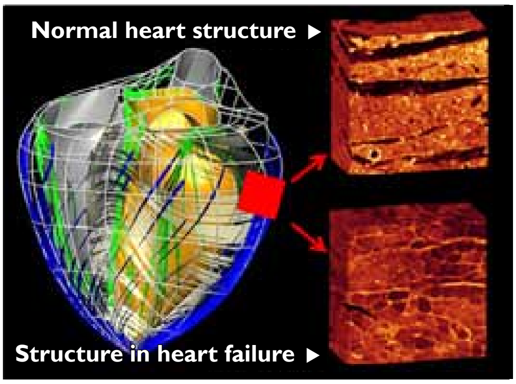
See also

www.abi.auckland.ac.nz/uoa/home/about/our-research/projects/biomechanics-for-breast-imaging

www.abi.auckland.ac.nz/uoa/home/about/our-research/projects/cardiac-mechanics

"Many mathematicians have a natural affinity with music. Mathematics departments invariably have little trouble assembling an orchestra from the ranks of their members."

Marcus de Sautoy, *Music of the Primes*



$$\frac{\partial \sigma^{ij}}{\partial x_i} + pb^j = pa^j$$

MATHEMATICAL EVENTS

June 24-28, 2013, Shanghai, China
Second Pacific Rim Mathematical Association (PRIMA) Congress, meeting.healife.com/prima2013/en/index.asp

October 1-4, Wellington
Absolutely Positively Mathematics & Statistics (NZ Assn of Maths Teachers Conference), nzamt13.org.nz

November 24-27, Hamilton
Joint Conference of the NZ Statistical Association and the Operations Research Society of NZ, smiller@stats.waikato.ac.nz or vanessa.cave@agresearch.co.nz

November 24-29, Kiama, NSW, Australia
Lighthouse DELTA 2013 (Conference on the teaching and learning of undergraduate mathematics and statistics), www.delta2013.net

December 2-4, Wellington
4th Wellington Workshop in Probability Theory and Mathematical Statistics, msor.victoria.ac.nz/Events/WWPMS2013

December 3-5, Tauranga
NZ Mathematical Society Colloquium nzmathsoc.org.nz/colloquium2013/home.php

January 13-17, 2014, Te Anau
NZMRI summer meeting (on Operator Algebras), www.maths.otago.ac.nz/nzmri14

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$$D_{n,n'} = \sup_x |F_{1,n}(x) - F_{2,n'}(x)|,$$

The statistics of wellbeing

Canterbury University PhD student Lisa Henley wants to picture the links between countries where people flourish and the values which enable them to do that.

"Gross domestic product (GDP) was developed in the USA to help production planning during World War II," says Henley, "and was never intended as a measure of national welfare. But GDP misses crucial parts of a country's wellbeing – those outside of monetary exchange."

Henley used free online global datasets about human wellbeing, flourishing and life satisfaction, which include up to 300 variables. She used a genetic algorithm and linear regression to select the most descriptive.

"These algorithms model natural selection and are a great way to find the best solution to a problem," says Henley. "You start with a population that represents your problem – various subsets of the 300 variables - constructed as binary elements."

"A binary tournament compares one set of variables to another and the fittest one progresses to the next level. All the winners are then resampled, swapping a length of their chromosome. The new chromosomes are tested for fitness, recombined with their parents, and winners proceed to the next level."

The algorithm repeats itself, calculating the average fitness of the result after each iteration. "Once it's been stable for 20 runs, it stops. I ended up with just over 100 variables, which is still a huge amount. The average years of secondary schooling for girls is coming through as really important."

The next step is to use genetic and clustering algorithms to find groups of countries according to levels of flourishing. "I'm using a multi-component fitness function, including some fairly cool non-parametric statistics to test the differences between the cluster populations."

Henley wants to represent the results visually, and may develop something similar to Gapminder software, which converts up to three variables over time into animated and interactive graphics. "Unfortunately, data available around inequality is still minimal."

When her data is graphed, she will look for "a happy people, happy planet 'sweet spot', and associate these with the values of countries. She will use data from the World Values survey to find "an ideal combination of values that lead to a flourishing life on a flourishing planet".

Henley says that much of the research into human flourishing "appears to be top-down, with variables being pre-defined. But statistics offers a wealth of methodologies for letting the data express the essence of a concept."

Henley attributes her interest in human flourishing to her time working in London as a research and development manager for Loyalty, the Sainsbury supermarket chain reward card, during the global bank crisis. There was a lot of discussion about whether buying more consumer goods was all there was to happiness, she says. "We think that increasing our wealth will increase our wellbeing, but the research shows that once we can meet our basic needs, there is a threshold after which further wealth doesn't increase our life satisfaction substantially."

See also

www.gapminder.org/
www.happyplanetindex.org/data/
www.prosperity.com/
www.tableausoftware.com/products/public

Notable maths problems

ERDÖS–FABER–LOVÁSZ CONJECTURE

If a graph (network) is made up of k complete subgraphs, each with k vertices, and having at most one vertex in common between any two of them, then the vertices of the graph can be coloured with k colours, such that adjacent vertices have different colours.

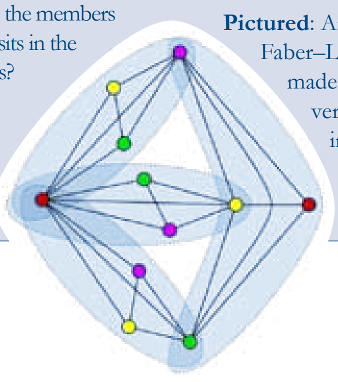
Simply: Suppose that an organisation has k committees (corresponding to complete subgraphs), each consisting of k members (corresponding to graph vertices), with any two committees having at most one member in common. All committees meet in the same room, which has k chairs (corresponding to vertex colours). Is it possible to assign the members to chairs so that each member sits in the same chair for all their meetings?

Discipline: Combinatorics, discrete mathematics

Originators: Hungarian mathematicians Paul Erdős and László Lovász and American mathematician Vance Faber formulated this innocent-sounding conjecture at a party in 1972, thinking it was trivial.

Incentive: When the authors realised the conjecture was harder than expected, Erdős offered US$50 for a proof that it was true, later raising this to US$500.

Progress: In 1988, W.I. Chiang and Eugene Lawler proved that the edge-colouring number of the graphs in the conjecture is at most $3k/2 - 2$, and four years later Jeff Kahn improved this to $k + o(k)$. In 2008, Abdón Sánchez-Arroyo established the truth of an equivalent version of the conjecture for dense hypergraphs.



Pictured: An instance of the Erdős–Faber–Lovász conjecture: A graph made up of four cliques of four vertices each, any two of which intersect in a single vertex, can be four-coloured.

How to stop an epidemic

The models built by mathematicians like Professor Mick Roberts, of Massey University at Albany, have saved many people from sickness and death.

He builds and analyses models of how infectious diseases spread in populations, to determine how pathogens can be controlled or eliminated.

"In 1996, I built a model of measles in New Zealand and analysed the current vaccination policy." The measles, mumps and rubella vaccine was then scheduled at 15 months and 11 years. "I showed that that was not going to prevent recurrent epidemics of measles; as a result the Ministry of Health changed the schedule to 15 months and four years."

"With whooping cough (which is caused by a bacterium) the result was the opposite," says Roberts. "An immunisation had to be added at age 11 because the vaccine protects for up to 10 years, unlike measles which gives protection for life."

"We get new viruses every year; for example, there are three seasonal influenza viruses, H1N1 and H3N2 - which are both influenza A - as well as influenza B." These viruses are slightly different each year. "Each flu season you get a mixture of all of them in different proportions."

"We get a pandemic when there's a major shift in a virus, leaving most people with very little protection." Roberts also helped with the ministry's pandemic planning for SARS. "In 2002, SARS had a four-day period between infection to becoming infectious, so there was time to do some contact tracing."

When exotic diseases enter society, health services need to know where to focus their efforts. "For example, if it's a point of entry for a region, you need to concentrate your resources there and model those networks so the spread of infection is minimised." What interests Roberts is the methodology behind the models.

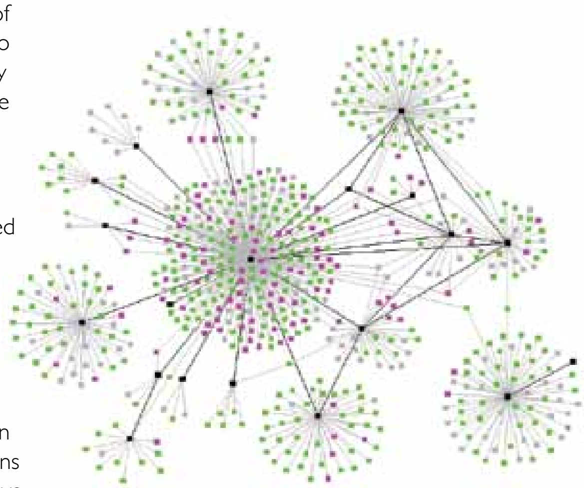
"The contact process is different for each infection – you're not going to get HIV from someone you're standing next to in a bus, whereas flu you could." Models describe how diseases spread through contact networks - based on sizes of households and workplaces and average numbers of people met in a day - and how the structure of these networks might change how infectious diseases spread.

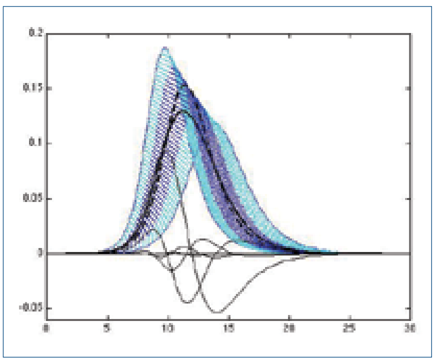
"I'm trying to improve the non-linear differential and integral equations behind the models. They're a highly complicated set of very big equations; we have to find ways to simplify them to something we can actually solve or understand. You can't always solve them but you can assess qualitatively what the solution might look like."

The crucial point is the threshold, where things change drastically. This is represented by the basic reproduction number, R_0. "If the number is greater than one, you can get an epidemic; if it's less you won't," says Roberts. Models for some diseases also have to represent contact between humans and animals; mosquito carriers in the case of dengue fever. He is working on more sophisticated and accurate predictions of the epidemic curve. "Predicting the curve tells us how many people will get sick next week or by the middle of winter. It forecasts the demand on health services."

Roberts' main collaborator for the last 20 years is Professor Hans Heesterbeek of Utrecht University in the Netherlands. Since they shared an office for six months at Cambridge University in the UK they have met almost every year, and their families have become good friends. "Our skills complement each other – we both have good intuition in different areas. The results we get are better than the sum of our individual abilities."

Below: Mapping the spread of contagion with contact tracing, © 2003 by Valdis Krobs, www.orgnet.com/contagion.html, used with permission.
Bottom: An epidemic curve.





$$R_0 + \frac{1}{p}\log\left(1 - \frac{N}{S(0)}p\right) = 0$$

Statistics and forensic evidence

Professor James Curran, of the University of Auckland, consults with forensic agencies in New Zealand, Australia and the UK, producing expert systems software to interpret crime evidence. He has also appeared as an expert witness about DNA and glass evidence in the USA and Australia.

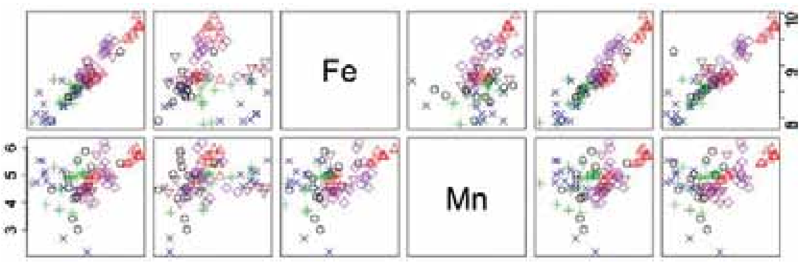
Curran's work has helped interpret evidence involving DNA, glass, mosquitoes, cellphone records and stolen pounamu. "Most law enforcement agencies spend their money on DNA, so that's the bulk of my work," he says. Globally, the number of forensic statisticians is small, as generally they need to be attached to a forensic agency to access crime scene data.

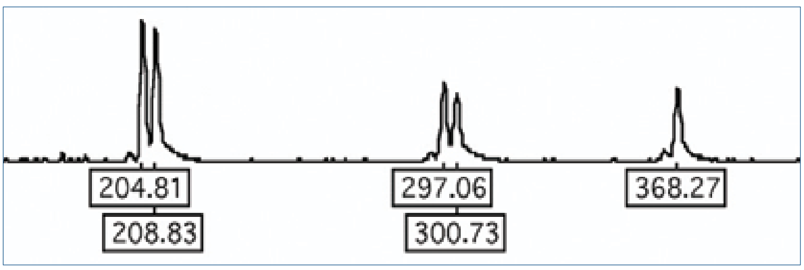
DNA evidence can be gathered from as little as ten cells, but it is often not as clear cut as juries expect from watching TV forensic shows, instead involving subjective interpretation and judgement calls. To interpret DNA, forensic scientists compare differences in the length of two alleles at standard places on the chromosome. "The position of the two peaks tell us the length variant, and their height reflects the amount of DNA in the sample," says Curran. "If it's all from one person, there should be clear spikes of the same height across the profile, but when there is small amount of DNA that may not be the case. We're trying to understand what controls this."

Forensic statisticians use standard statistical tools such as log-linear modelling and Bayesian estimation to model the stochastic behaviour of evidence. The weight of the evidence is presented in court using a likelihood ratio.

Curran gives an example of a break-in where the burglar cut himself and left a blood stain, and the suspect's DNA matches the stain. "The statistician has to calculate the likelihood of the match if the suspect left the blood at the scene, or someone unrelated left the blood. The likelihood ratio is the ratio of these two quantities. Using standard population genetic models, they can say the evidence is 3.8×10^{13} times more likely if the suspect left this blood rather than if someone unrelated left the blood."

Samples of mixed DNA from two people are common in rape cases. "The bulk of the DNA is usually from the victim and smaller peaks are from the offender," says Curran. These profiles have up to four peaks at each locus. There are six different ways that four peaks may be assigned to two people. This increase in the number of possibilities weakens the strength of the evidence.





"Many profiles have small bumps next to the peaks, called stutters. In a mixed sample, the stutter peak could be confused with the offender's allele." This confusion also weakens the strength of the evidence, and complicates the interpretation.

Glass is a common type of trace evidence, and this field is another heavy user of statistics. Fragments found on suspects are often smaller than a grain of salt; the most common technique compares their refractive index with a sample of control glass from the crime scene. The chemical composition of glass, or any biological or inorganic substance, can also be measured and used to associate evidence with a crime scene or not. One of Curran's recent projects involved a case where a cellphone call was made to a companion in the getaway car by a Dutch bank robber trying to fake an alibi.

"The evidence was a voice echo on the line," he said. "The Netherlands people estimated from their experiments that the echo was about 42 times more likely if the call was made inside a car rather than outside. But they were concerned that this statement was stronger than it should be. Our research concluded that the evidence was about 12 times more likely."

One of Curran's students used multivariate statistics to describe the elemental composition of greenstone from different locations. The research arose from Operation Roar, after which two helicopter pilots were convicted of the theft of 20 tonnes of pounamu. "Old school geologists, used to judging by shape and colour, said we couldn't differentiate between different sources of greenstone using elemental techniques," he says.

"Questions in the interpretation of evidence are primarily probabilistic; we don't know exactly what happened and have to speculate," he says. "Statistics is involved in the same way in most sciences, dealing with variation, and modelling in the face of uncertainty."

Above: Iron and manganese in different outcrops of Wakatipu pounamu.
Left: An electropherogram showing tiny DNA stutter peaks to the left of the main peaks.

Mathematical poetry and tables

Dr Clemency Montelle, of Canterbury University, studies ancient mathematical texts in dead languages, combining her love of both fields.

"I took Greek, Latin and maths at uni because I enjoyed them at school," she said. "After four years, I came across a book on Euclid's elements in the original Greek and realised I could read it and understand the maths, and I was really interested in figuring out what mathematical texts in ancient languages meant."

Sanskrit was the next step. "Then I learnt classical Arabic and picked up Akkadian. Once you have a couple of languages under your belt you know the rubric for learning them," she says. Akkadian emerged in ancient Mesopotamia (modern Iraq) around 3,600 BCE* and was written on clay tablets. "The first of these were found only 150 years ago - it was the first mathematically literate culture."

Montelle says that the European pioneers of mathematical history neglected the arts of 'number crunching', such as the construction of function tables and algorithms, particularly in non-Western traditions such as the Sanskrit sciences of India between 900 and 1800CE**. There are very few historians working in this area, in or outside India.

She is part of three projects studying these ancient Sanskrit mathematical manuscripts. One is *Histoire des tables numériques,* a four-year international study led by Dominique Tournès of Réunion in the Indian Ocean. Montelle has also received a Marsden Grant to identify, catalogue and publish online all known Sanskrit table texts, with Kim Plofker, of Union College in New York. They will also critically edit three key unpublished texts of tables. Montelle also received a 2012 Rutherford Discovery Fellowship to study a wider selection of Sanskrit mathematical documents.

"There might be 25 different manuscript copies of one text. We want to collect as many copies as possible, translate and compare them and write a commentary," she says. Many technical words, such as hypotenuse, aren't in Sanskrit dictionaries, "so we have to determine them from the context".

Tables are so commonplace now that their significance is almost lost on modern mathematicians, she says. "They're the most valuable source for seeing how mathematicians actually computed things. We can read their theoretical ideas about the position of the sun, but the only place we see them calculating it is in tables." The pair will examine mostly astronomical tables, about eclipses, planetary motions, astrology and trigonometry.

Montelle "spent two summers at the Houghton Library of rare books at Harvard, cataloguing their 2,500 Sanskrit documents, because nobody could read Sanskrit." Many manuscripts are in India and also not catalogued, so she will be working with the Chennai Mathematical Institute and visiting libraries there.

"Indian mathematics was composed into verse that was meant to be memorised," says Montelle. "Quite technical mathematics and scientific ideas were fitted into memorable poetry. Long and complex numbers – say pi to ten decimal places - become more of a challenge when they have to be expressed in verse."

Rather than trying to find a geometrical demonstration of astronomical events, using careful proofs as in the European tradition, Indian mathematicians were more interested in the algorithms for computing the events, she says. "They had several ways to compute the position of the sun and tolerated multiple models, while the West was fixated on one unified solution."

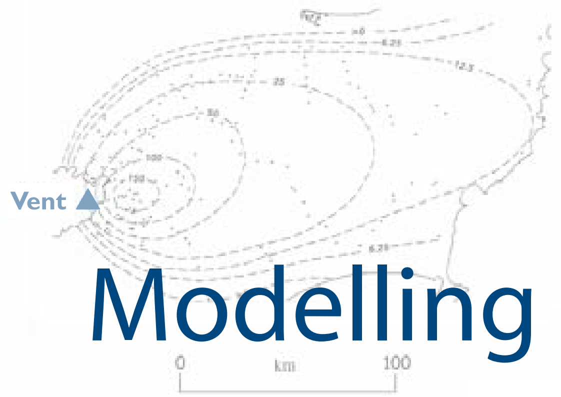
The projects will help explain how Indian astronomers developed new models, and how mathematical knowledge was transferred between the two traditions.

"As we explore complex areas such as quantum physics and coding theory for computers, science is becoming heavily computational and hearkens back to the emphases of Indian mathematicians," she says.

*BCE: Before the common era
*CE: Common era

Below: A copy of a manuscript in Sanskrit with a numerical table.

Inset: A close-up including the first 10 numbers. The place value base ten system of numeration – the use of 10 different glyphs to represent a number of any magnitude - first appeared in India. These are the beginnings of the numerals we use today.



$$c(x,t)=\frac{Q}{\sqrt{4\pi Dt}}e^{-\frac{(x-Ut)^2}{4Dt}}$$

Modelling **pollution** and **plumes**

Professor Robert McKibbin, of Massey University at Albany, models the transport of groundwater pollution and volcanic plumes in the atmosphere using very similar differential equations.

Waste from industry or agriculture commonly pollutes groundwater aquifers in most countries, says McKibbin. "The challenge is to find good ways of predicting where the pollutant will go."

One example of a pollutant was "sea water intrusion into fresh water aquifers near the coast in Queensland". There is a similar concern that drawing hot water out at Waiwera may be allowing sea water to seep into underground aquifers. However, his models are not based on one pollutant or geographical area, but are generic and meant to apply anywhere.

Aquifers are a kind of porous media, a structure containing many fine pathways, with a pressure difference across distance. "There has been a lot of work done on soluble pollutants over decades," he says, using standard computer packages for computational fluid dynamics. However, they use time-consuming numerical simulations.

"Because we can't see into the ground, and often can't drill cores, there are always unknown parameters in these models, and they have to be estimated."

"We're looking for more simplified models that are quicker and still accurate. They are semi-analytic models, using simpler differential equations to find simple solutions which can be written as formulae." PhD student Amjad Ali is modelling groundwater aquifers with different rock layers to improve the complexity of the model.

Their solutions were tested against experimental data from a site in Canada, where chloride ions, acting as a soluble tracer, were injected into a well and downstream measurements from the aquifer were monitored over a long period. "We found the data was a good fit with our model, which meant we could have told them what would happen next."

Modelling the flight of particles in the atmosphere is another generic problem using similar differential equations, which can be applied to volcanic ash, hydrothermal eruptions, dust, smoke and sand clouds, pollen, and horticultural spray drift. "Large particles fall fast, but tiny ones tend to stay high in the air and blow a long way. The best local examples are smoke from Australian bushfires, where the dropping speed is almost negligible, and volcanic plumes from the central North Island. Organisations managing natural disasters need to know where that ash is going to fall."

Their model predicted the same shape as measured data from historic Taupo eruptions, where the thickness of drill core layers has been plotted on maps. "Volcanologists are keen to know what's going on in volcanic plumes, so we're now trying to predict the release height of particles in the plume."

PhD student Sharleen Harper, now working for NIWA, used the same techniques to calculate the way that spray droplets spread after being released in an orchard.

"There's enormous commercial investment in computer programs used to calculate movement of groundwater pollution," says McKibbin. "They use the same the differential equations we do. They're not wrong, but we're investigating less costly ways to solve those equations. The data from underground is pretty scarce anyway. Complicated models are not particularly useful when you have only a few bits of data."

Top: A model of the thickness of deposits of volcanic fragments - tephra - from a Taupo eruption.

Awards and honours

Roy Kerr (University of Canterbury); the 2013 Einstein Medal for his 1963 solution to Einstein's gravitational field equations.

Rod Downey (Victoria University of Wellington); 2011 Hector Medal from the Royal Society of NZ.

Robert McKibbin (Massey University); 2012 ANZIAM Medal (Australian and NZ Industrial and Applied Mathematics).

Rob Goldblatt (VUW); 2012 Jones Medal from the RSNZ.

James Russell (University of Auckland); 2012 NZ Prime Minister's Prize for Young and Emerging Scientists.

Adam Day (VUW); 2011 Sacks Prize for the best PhD thesis in logic and 2011 Hatherton Award from the RSNZ.

James Sneyd (UA); 2014 Maclaurin Lectureship by the American and NZ Mathematical Societies.

Clemency Montelle (UC); a five-year Rutherford Discovery Fellowship.

Noam Greenberg (VUW); a five-year John Templeton Turing Research Fellowship.

Marston Conder (UA), **Rod Downey** (VUW), **Vaughan Jones** (UA and Vanderbilt University) and **Gaven Martin** (Massey University); inaugural Fellows of the American Mathematical Society.

A special mention for pure and applied mathematics, which gained the highest average quality score (5.81) of all subjects in the latest Performance Based Research Fund evaluation, with 31.50 As (26.5%) and 50.23 Bs (42.2%). Congratulations to all who contributed to this outstanding achievement!

The orientation in space of the principal stresses in the San Fernando Valley in California.

Fisheries and tectonic stresses

Dr Richard Arnold, of Victoria University, is using statistical methods to estimate the size of New Zealand fisheries, and tectonic stresses in earthquake zones.

No one knows the total harvest, combining commercial, recreational, customary and illegal fishing, from any fishery area or stock in New Zealand, says Arnold.

In 2011, Ngāti Kahungunu wanted to know how much was being taken from the waters in their region, and whether their people were using recreational or customary licences to fish. Arnold and Masters student Kylie Maxwell, with the help of the iwi and the Ministry of Fisheries, collated the total harvest of 10 key fish species in the iwi area between 2007 and 2010.

Maxwell obtained commercial catch data from the ministry, and the results of recreational fishing surveys at local boat ramps. She obtained permission to study the records of 41 Māori fisheries officers, who issue customary permits for the coast between Wairoa and Cape Palliser. She also counted illegal fishing catches from the Ministry of Fisheries' conviction database.

Results showed that Ngāti Kahungunu people were largely using recreational fishing rights to fish for rock lobster, pāua and kina. Customary licences were often used for special events "when they might want a lot of pāua at once", says Arnold. Maxwell's estimate of total catch and species were illustrated in heat maps of the ocean showing the size of the catch across the Ngāti Kahungunu area, and pointed to a need for more consistent and higher quality data, especially on the recreational catch.

Antarctic toothfish

Arnold worked with a PhD student, Darcy Webber, to build a statistical model of the total Antarctic toothfish population, using commercial catch data. This fish, which has only been caught commercially since 1997, lives for at least 50 years, growing up underneath inshore sea ice and migrating further from shore later in life. "If you don't take that migration into account when you estimate the population" says Arnold, "you can seriously overestimate the size of it."

Webber integrated commercial catch data with scientific trawling surveys where fish are tagged, released and then later recaptured by the commercial fishery. The standard multi-stage statistical regression models work backwards from landed catch to how many fish there are in the sea, based on many assumptions. "We needed to examine all the assumptions in these models, including how fish grow, how well-mixed and distributed fish populations are in the sea, and how efficient fishing gear and fishing techniques are in capturing the fish."

The study is not finished, but Webber's initial estimate is that the toothfish population has dropped by approximately 15 percent since commercial fishing began, in contrast to some perceptions that commercial fishing has devastated the species.

Tectonic stress

In an earthquake the tectonic stresses in the earth overwhelm a planar weakness and two blocks of rock slide against each other. Arnold, with collaborator John Townend, has developed new methods for estimating the likely orientations of these earthquake fault planes.

"The statistical problem is that you get a very fuzzy view of each earthquake – the signal observed at seismometer stations reveals the geometrical properties of the earthquake, but it is wrapped in seismological noise added as the wave travels. Estimating the fault orientation and direction of the slip along the fault is difficult - you have to separate the signal from the noise."

"Solving this problem needed a new branch of directional statistics, of geometrical objects oriented in space that can have the multiple reflectional symmetries that earthquake fault planes do. My work with collaborator Peter Jupp has introduced new statistical methods that allow us, for example, to test whether the dominant stress is horizontal."

Arnold used the new methods to test whether the recent major earthquakes in Christchurch had changed the pattern of tectonic stress around the city. "Between the September 2010 and February 2011 major earthquakes the average location of the smaller earthquakes moved steadily eastward, but we found that there hasn't been any change in the orientation of the underground stresses driving those earthquakes."

$$\tan(2\emptyset^{m}_{G}) = \frac{2[R_{11}R_{21} + vR_{12}R_{22}]}{(R_{11}^2 - R_{21}^2) + v(R_{12}^2 - R_{22}^2)}$$