IMAges NZ

ISSUE TWELVE

# The ultimate interactive machines

**Imagine walking into a building where wall-sized computer-generated faces greet you by name and interact with you as thinking and emotional beings; imagine seeing images of the brain activation generating those expressions. This is one of the visions double Academy Award winner Mark Sagar has for his forthcoming Laboratory for Animate Technologies at the University of Auckland.**

Famous for his work on facial animation and rendering systems used in movies such as *King Kong*, *Spiderman 2* and *Avatar*, Sagar moved from his job as Special Projects Supervisor at Weta Digital in Wellington for the intellectual and collaborative opportunities available in a university.

In film, he went from capturing animation to simulation, and now he wants "drive the virtual nerves themselves. There's a big difference from pre-recording performances for films to creating naturalistic behaviour from core principles."

Sagar is establishing funding, developing foundation software and discussing collaborations that will make the lab a reality in 2013. A discussion on where the lab walls would go was audible in the background when he spoke with IMAges.

The lab will carry out blue skies research on interactive computational intelligence as well as applied research with a raft of disciplines. This could include responsive architecture – buildings that respond interactively with their users; healthcare robotics –caring machines with friendly, emotionally responsive faces; education – encouraging the retention of information in interactive computer learning. "It could be anything where you want to create an emotional connection between the system and participants, more of a natural connection with technology," he says.

He plans to collaborate with several arts and science research groups from the University of Auckland and AUT, including the Centre for Brain Research and the Knowledge Engineering and Discovery Research Institute, "embodying and creating interactive neural models of current theories of how the brain processes data", as well as helping to create emotional models of synthesised computational speech.

His field uses a huge range of mathematics. "There's lots of computational geometry - models are typically built out of geometric patches. For example, finite element method combines small patches of a model into a whole which can represent quite complex forms. Constitutive equations define how a model elastically deforms in response to forces generated by muscles in simulations. We use lots of optimisation and numerical analysis. We also use computational intelligence and computer vision, and neural networks. Each aspect uses a different type of mathematical model."

The lab will have a small research team developing the core software framework, with more working on targeted applications, including arts and science students. He is interested in mathematics students working with computer modelling and spatiotemporal analysis.

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## Welcome

This twice-yearly IMAges bulletin was previously published by the New Zealand Institute of Mathematics and its Applications (the NZIMA).

The NZIMA was formally disestablished in March 2012 (after its CoRE funding ran out), but this bulletin will continue to be published with the support of New Zealand universities.

We hope you enjoy this issue. It features a range of items about some of the postgraduate students supported by the NZIMA over the last ten years -- what did they achieve, and where are they now?

Please let us know if you have any requests or suggestions for items in future issues.

**Marston Conder and James Sneyd**
**Editors, University of Auckland**

F=ma

## INSIDE

Sagar has the maths/art background that synthesises animated bodies and computers – his father was a systems analyst and his mother an artist. He helped support himself while travelling by painting portraits of tourists in China, the UK and Nepal. For his Masters, he built a 3-D computer model of the human eye, and in his PhD he wrote the software for people to build biologically accurate computer models of complex human anatomy.

When Hollywood called at MIT for techies to make computer-synthesised actors, his software already created computer-generated and manipulable features from dots on a person's face. The animation he and Paul Charette built asked "Am I real or am I digital?" and led to his career with Imagewords, LifeFX and Weta.

When asked to look ten years into the future of the field, he expects interactive holograms to be available and "movies like Avatar will be real-time experiences where you interact emotionally with the characters in your living room." However, he says, the lab will "initially focus not so much on the big concepts, but on the small details that make those things convincing".

$$\min \|Ax-b\|_2^2 \quad a \le x \le B,$$

> Mathematics is an edifice, built upon axioms, in which a theorem proved in ancient Greece is still a theorem in twenty-first century mathematics.
>
> *Marcus de Sautoy*, Music of the Primes.

> The highest moments in the life of a mathematician are the first few moments after they have proved the result - just before they find the mistake.
>
> *Anonymous*

## NOTABLE MATHS PROBLEMS

### ARE THERE INFINITELY MANY PERFECT NUMBERS?

**Simply:** A positive integer $n$ is a perfect number if it is equal to the sum of all of its positive divisors, except for $n$. For example, 6 is the first perfect number because 1+2+3=6. The first four perfect numbers were known before 100 AD. Most number experts think there are infinitely many, but there is no proof. And no odd perfect numbers are known at all.

**Discipline:** Number theory.

**Progress:** around 300BC, Euclid proved that if $2^p-1$ is prime, then $2^{p-1}(2^p-1)$ is perfect, and in the 1700s, Euler proved that every even perfect number has this form. Prime numbers of the form $2^p-1$ are called Mersenne primes (after the 17th century French monk Marin Mersenne), but it is still not known if there are infinitely many.

The Canadian mathematician Donald Gillies used statistical theory in 1964 to argue (but not prove) that there are infinitely many even perfect numbers. Since 1996, the Great Internet Mersenne Prime Search (GIMPS) has harnessed idle computer power to hunt for Mersenne primes. The 35th to the latest, the 47th known Mersenne prime, were discovered as part of GIMPS.

**Incentives: In 2008,** GIMPS won the $100,000 Cooperative Computing Award for the 45th Mersenne prime, which has 12,978,189 digits. There is a $150,000 prize for the first hundred-million digit prime, and a $250,000 prize for the first billion-digit prime.

**Contributions:** The search has led to new and faster ways of multiplying large integers; prime programmes are regularly used to test computer processing units such as the Pentium II and Pro chips, and identified the Pentium bug.

**Unusual aspects:** Except for 6, every even perfect number leaves remainder 1 when divided by 9.

Every even perfect number is represented in binary as $p$ ones followed by $p$-1 zeroes: 6=110, 28=11100, 496=111110000, and 8128=1111111000000.

The University of Illinois mathematics department changed their postage meter, below, to reflect their discovery of the 23rd Mersenne prime in 1963.


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# Random binary sequences

**Adam Day's whole mathematics career has been about two numbers, the 0 and 1 of infinite binary sequences. The new field of algorithmic randomness in which he works defines a sequence as random if it appears so to any algorithm.**

The field captures three intuitions about randomness; the first being that random numbers are complex and incompressible. A complexity function assigns a natural number to each finite binary sequence, representing the complexity of the sequence, A sequence is considered random if its complexity is longer than its length. For example, the binary sequence 01001 has length 5, so if its complexity was 4 it would be considered not random. Infinite sequences can also be seen as sets of natural numbers.

The second intuition is that random numbers should pass statistical tests for randomness, and the third is that no betting strategy on any computer should be able to make money betting on a random sequence.

The field draws from measure and computability theory, which abstracts computing to an environment with no restrictions on memory or time, and uses idealized oracle machines to study decision problems. Algorithmic randomness has applications to random number generators based on quantum mechanics, but like many other areas of pure mathematics it is largely applied to other areas of the discipline such as logic.

Day's PhD thesis at Victoria University of Wellington, supervised by Rod Downey and Noam Greenberg, developed new results unifying previously separate Russian and European developments in the field. It won him the 2011 Royal Society of NZ Hatherton Award for the best scientific paper by a PhD student at a New Zealand university, and was one of the two 2011 winners of the international Association for Symbolic Logic's Sacks Prize.

He was also awarded a three-year Miller Research Fellowship at the Berkeley campus of the University of California, which goes to "exceptional young scientists". "I would never have anticipated being here five years ago," he said from Berkeley. Originally, he returned from working overseas to study as a secondary mathematics teacher. "I was going to do a year of maths then a year at teachers' college – I just started doing some maths papers and never stopped."

He eventually hopes to get a permanent university mathematics job in New Zealand, but is now "trying to learn as much as I can from the experts here on the relationship between set theory and randomness".

Answering one question just leads onto another, he says. "For example, lots of mathematics theorems have a statement such as 'For almost all the real numbers some property x is true'. I've been looking at to what extent you can capture that notion of 'almost all' in algorithmic randomness."

In particular, Day is interested in ergodic theory, the study of the behaviour of dynamical systems over a long period of time. His current question is "does a random particle always comes back to itself"?

For Day, mathematics has realms of gold just like those Keats wrote about in his poem on Chapman's translation of Homer. Like literature, mathematics is done entirely in the mind; it has "amazing sites and beauties – it's just a privilege to travel round this most coherent body of logical thought, contemplating it and adding your own contribution".

**Adam with his son Peter learning about American geography in their Berkeley apartment.**