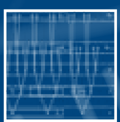
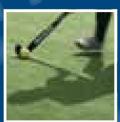
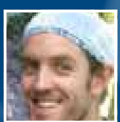
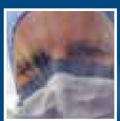
IMAges NZ
ISSUE TWELVE

INSIDE









The ultimate interactive machines

Imagine walking into a building where wall-sized computer-generated faces greet you by name and interact with you as thinking and emotional beings; imagine seeing images of the brain activation generating those expressions. This is one of the visions double Academy Award winner Mark Sagar has for his forthcoming Laboratory for Animate Technologies at the University of Auckland.

Famous for his work on facial animation and rendering systems used in movies such as *King Kong*, *Spiderman 2* and *Avatar*, Sagar moved from his job as Special Projects Supervisor at Weta Digital in Wellington for the intellectual and collaborative opportunities available in a university.

In film, he went from capturing animation to simulation, and now he wants "drive the virtual nerves themselves. There's a big difference from pre-recording performances for films to creating naturalistic behaviour from core principles."

Sagar is establishing funding, developing foundation software and discussing collaborations that will make the lab a reality in 2013. A discussion on where the lab walls would go was audible in the background when he spoke with IMAges.

The lab will carry out blue skies research on interactive computational intelligence as well as applied research with a raft of disciplines. This could include responsive architecture – buildings that respond interactively with their users; healthcare robotics –caring machines with friendly, emotionally responsive faces; education – encouraging the retention of information in interactive computer learning. "It could be anything where you want to create an emotional connection between the system and participants, more of a natural connection with technology," he says.

He plans to collaborate with several arts and science research groups from the University of Auckland and AUT, including the Centre for Brain Research and the Knowledge Engineering and Discovery Research Institute, "embodying and creating interactive neural models of current theories of how the brain processes data", as well as helping to create emotional models of synthesised computational speech.

His field uses a huge range of mathematics. "There's lots of computational geometry - models are typically built out of geometric patches. For example, finite element method combines small patches of a model into a whole which can represent quite complex forms. Constitutive equations define how a model elastically deforms in response to forces generated by muscles in simulations. We use lots of optimisation and numerical analysis. We also use computational intelligence and computer vision, and neural networks. Each aspect uses a different type of mathematical model."

The lab will have a small research team developing the core software framework, with more working on targeted applications, including arts and science students. He is interested in mathematics students working with computer modelling and spatiotemporal analysis.

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Welcome

This twice-yearly IMAges bulletin was previously published by the New Zealand Institute of Mathematics and its Applications (the NZIMA).

The NZIMA was formally disestablished in March 2012 (after its CoRE funding ran out), but this bulletin will continue to be published with the support of New Zealand universities.

We hope you enjoy this issue. It features a range of items about some of the postgraduate students supported by the NZIMA over the last ten years -- what did they achieve, and where are they now?

Please let us know if you have any requests or suggestions for items in future issues.

Marston Conder and James Sneyd
Editors, University of Auckland

F=ma

◀ I Sagar has the maths/art background that synthesises animated bodies and computers – his father was a systems analyst and his mother an artist. He helped support himself while travelling by painting portraits of tourists in China, the UK and Nepal. For his Masters, he built a 3-D computer model of the human eye, and in his PhD he wrote the software for people to build biologically accurate computer models of complex human anatomy.

When Hollywood called at MIT for techies to make computer-synthesised actors, his software already created computer-generated and manipulable features from dots on a person's face. The animation he and Paul Charette built asked "Am I real or am I digital?" and led to his career with Imagewords, LifeFX and Weta.

When asked to look ten years into the future of the field, he expects interactive holograms to be available and "movies like Avatar will be real-time experiences where you interact emotionally with the characters in your living room." However, he says, the lab will "initially focus not so much on the big concepts, but on the small details that make those things convincing".

$$\min\|Ax-b\|_2^2 \quad a \le x \le B,$$

> Mathematics is an edifice, built upon axioms, in which a theorem proved in ancient Greece is still a theorem in twenty-first century mathematics.
>
> *Marcus de Sautoy*, Music of the Primes.

> The highest moments in the life of a mathematician are the first few moments after they have proved the result - just before they find the mistake.
>
> *Anonymous*

NOTABLE MATHS PROBLEMS

ARE THERE INFINITELY MANY PERFECT NUMBERS?

Simply: A positive integer *n* is a perfect number if it is equal to the sum of all of its positive divisors, except for *n*. For example, 6 is the first perfect number because 1+2+3=6. The first four perfect numbers were known before 100 AD. Most number experts think there are infinitely many, but there is no proof. And no odd perfect numbers are known at all.

Discipline: Number theory.

Progress: around 300BC, Euclid proved that if 2^p-1 is prime, then $2^{p-1}(2^p-1)$ is perfect, and in the 1700s, Euler proved that every even perfect number has this form. Prime numbers of the form 2^p-1 are called Mersenne primes (after the 17th century French monk Marin Mersenne), but it is still not known if there are infinitely many.

The Canadian mathematician Donald Gillies used statistical theory in 1964 to argue (but not prove) that there are infinitely many even perfect numbers. Since 1996, the Great Internet Mersenne Prime Search (GIMPS) has harnessed idle computer power to hunt for Mersenne primes. The 35th to the latest, the 47th known Mersenne prime, were discovered as part of GIMPS.

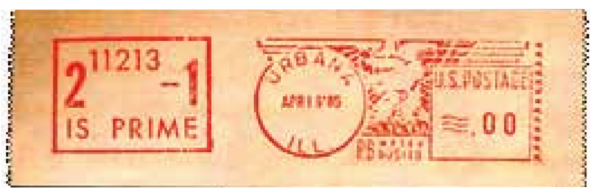
Incentives: In 2008, GIMPS won the $100,000 Cooperative Computing Award for the 45th Mersenne prime, which has 12,978,189 digits. There is a $150,000 prize for the first hundred-million digit prime, and a $250,000 prize for the first billion-digit prime.

Contributions: The search has led to new and faster ways of multiplying large integers; prime programmes are regularly used to test computer processing units such as the Pentium II and Pro chips, and identified the Pentium bug.

Unusual aspects: Except for 6, every even perfect number leaves remainder 1 when divided by 9.

Every even perfect number is represented in binary as *p* ones followed by *p*-1 zeroes: 6=110, 28=11100, 496=111110000, and 8128=1111111000000.

The University of Illinois mathematics department changed their postage meter, below, to reflect their discovery of the 23rd Mersenne prime in 1963.



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Random binary sequences

Adam Day's whole mathematics career has been about two numbers, the 0 and 1 of infinite binary sequences. The new field of algorithmic randomness in which he works defines a sequence as random if it appears so to any algorithm.

The field captures three intuitions about randomness; the first being that random numbers are complex and incompressible. A complexity function assigns a natural number to each finite binary sequence, representing the complexity of the sequence, A sequence is considered random if its complexity is longer than its length. For example, the binary sequence 01001 has length 5, so if its complexity was 4 it would be considered not random. Infinite sequences can also be seen as sets of natural numbers.

The second intuition is that random numbers should pass statistical tests for randomness, and the third is that no betting strategy on any computer should be able to make money betting on a random sequence.

The field draws from measure and computability theory, which abstracts computing to an environment with no restrictions on memory or time, and uses idealized oracle machines to study decision problems. Algorithmic randomness has applications to random number generators based on quantum mechanics, but like many other areas of pure mathematics it is largely applied to other areas of the discipline such as logic.

Day's PhD thesis at Victoria University of Wellington, supervised by Rod Downey and Noam Greenberg, developed new results unifying previously separate Russian and European developments in the field. It won him the 2011 Royal Society of NZ Hatherton Award for the best scientific paper by a PhD student at a New Zealand university, and was one of the two 2011 winners of the international Association for Symbolic Logic's Sacks Prize.

He was also awarded a three-year Miller Research Fellowship at the Berkeley campus of the University of California, which goes to "exceptional young scientists". "I would never have anticipated being here five years ago," he said from Berkeley. Originally, he returned from working overseas to study as a secondary mathematics teacher. "I was going to do a year of maths then a year at teachers' college – I just started doing some maths papers and never stopped."

He eventually hopes to get a permanent university mathematics job in New Zealand, but is now "trying to learn as much as I can from the experts here on the relationship between set theory and randomness".

Answering one question just leads onto another, he says. "For example, lots of mathematics theorems have a statement such as 'For almost all the real numbers some property x is true'. I've been looking at to what extent you can capture that notion of 'almost all' in algorithmic randomness."

In particular, Day is interested in ergodic theory, the study of the behaviour of dynamical systems over a long period of time. His current question is "does a random particle always comes back to itself"?

For Day, mathematics has realms of gold just like those Keats wrote about in his poem on Chapman's translation of Homer. Like literature, mathematics is done entirely in the mind; it has "amazing sites and beauties – it's just a privilege to travel round this most coherent body of logical thought, contemplating it and adding your own contribution".

Adam with his son Peter learning about American geography in their Berkeley apartment.

From anatomy to water

For her NZIMA-funded Masters, Kim Archibald (or Noakes as she was then) made one of the first three-dimensional anatomically-based computational models of the human pelvic floor.

The project aimed to benefit sufferers of fecal incontinence, something Archibald thinks "is really worthwhile even though it's not dinner-table conversation".

"Within medicine there wasn't a great understanding of the mechanisms of incontinence – there are often several contributing factors. People don't die of it, but it makes huge problems in their lives, and most are too embarrassed even to tell their doctor."

Developing the model was complex – "even surgeons struggle to identify on an image where different muscles start and stop. We spent hours looking at preserved tissue, cadavers, MRI scans and slides from the Visible Human Project (VHR) to develop three 3-D models – a male and female model based on the VHR slides and a female model based on MRI images of a healthy female volunteer."

"We used axial (horizontal), sagittal (left and right) and coronal slices (front and back), placed dots around the edges of each anatomical element, and used mathematical fitting procedures to create a mesh connecting all the dots. It took months to triple-check all the edges."

Then she used kinematics to model two transformations, a bearing down push in the Levator Ani muscle and squeeze pressure in the anal canal. Shannon Li has since used the model in her PhD model of childbirth, and another Auckland researcher is using it to investigate incontinence related to childbirth. "There's loads of future work to be done, this just set it up. If I did a PhD, I'd do it on this."

Archibald handed in her thesis the day before the national hockey trials in 2007, after which she was selected into the Blacksticks team. She juggled part-time work at the Auckland Bioengineering Institute with international hockey commitments, and in 2008 was a member of the New Zealand hockey team at the Beijing Olympics.

Afterwards, she played a semi-professional season with Hockey Club Rotterdam and juggled this with her first engineering job outside the university. She managed projects for Crosslinks, a small Dutch bioinformatics data analysis company, including a European Union project on mood disorders.

Back in New Zealand, she is an optimization engineer with Derceto, a local company which customises its Aquadapt software to save electricity costs for water utilities which have to use pumping stations to distribute water. This includes Greater Wellington Water and companies in Australia, North America, South Korea and the UK. The work includes detailed feasibility studies, design, implementation and support.

All her projects have used programmes that "do a lot of the maths for you, although you have to understand it. If Derceto didn't have the supercomputers, we'd spend hundreds of years trying to solve one equation."

"I did bioengineering because I really like the idea of improving people's quality of life. My cousin was born with spina bifida and I've seen how technology has helped him interact with the world, so anything that can alleviate problems like that is really good."

MATHEMATICAL EVENTS

29-30 November 2012, Dunedin
NZ Statistical Association 2012 Conference
www.maths.otago.ac.nz/nzsa2012/

4-6 December, Palmerston North
2012 NZ Mathematical Society Colloquium
http://nzmathsoc.org.nz/colloquium/home.php

4-7 December, Queenstown
Australasian Applied Statistics Conference 2012
www.aasc2012.com/

10-14 December 2012, Sydney, Australia
36th Australasian Conference on Combinatorial Mathematics and Combinatorial Computing
http://conferences.science.unsw.edu.au/36accmcc/

3-7 February 2013, Newcastle, Australia
ANZIAM'13: Annual Conference of ANZIAM
http://anziam2013.newcastle.edu.au/

24-28 June 2013, Shanghai, China
Second Pacific Rim Mathematical Congress
www.primath.org/prima2013/

1-4 October, Wellington
NZAMT Conference 2013: Absolutely Positively Mathematics and Statistics
See www.nzamt.org.nz/component/content/article/1-latest-news/171-nzamt-conference-2013

Optimising fleets around the world

Eyal Loz was overseas when he heard he had won a postgraduate scholarship in the NZIMA combinatorics programme – "without the scholarship I would have done my PhD overseas instead of here". He went on to do a PhD with a Top Achiever Doctoral scholarship.

His thesis on the degree diameter problem combined theoretical investigation with the design of original computer programmes in C++. He used covering and voltage graph techniques to construct many of what are now the largest known connected regular graphs of given degree d and diameter k, for d up to 20 and k up to 10.

During his PhD he also worked on a project with Tanglin Consultancy for Pacific Horizon Motorhomes. "They have bases all around the country and needed to schedule vehicles as bookings come in. The company was doing this manually, and had no way of analyzing data."

"Campervans are expensive - if you are running low, you can't buy one as easily as car rental companies can, so it is important to optimise usage of your fleet, and to deal with running out of vans in one base. I developed a component of a large application which automates and optimizes booking allocations. It improved their overall profitability, and can reduce costs by 20 percent."

Loz was able to use code from his PhD work. "Sometimes people don't understand how practical maths can be. I was finding very large graphs for particular problems which might have no applications. But the code you write, the theories you develop, and the problems you solve, all have applications. Most people completely miss this about high level mathematics."

He finished his PhD at the University of Auckland and worked as a C++ developer in the financial industry in Sydney, building trading platforms. He then worked as an assistant trader for Susquehanna International Group. "SIG recruits people with backgrounds in maths, computer science and physics - every trader there has a quantitative science degree. Options trading is extremely mathematical - you need to understand and apply models in a dynamic environment."

He is now working with partners from Tanglin on a startup software service called Sort-it, "a system to manage and optimize rental fleets for the tourism industry. It can take bookings online 24/7 and everything is optimized." The model is different from their earlier one, because it is designed to integrate easily with different systems. "Usually this kind of technology is completely outside of what smaller businesses can afford, but we want it to work for small and large firms."

The New Zealand company is not just aiming at the local market, but hopes to sell the software internationally.

See www.sort-it.co.nz/

Loz with Kakashi, the rainbow lorikeet he adopted when she fell out of a tree as a fledgling.

Causal effect in clinical trials

Jean Zhaojing Gong says that without her NZIMA doctoral scholarship she would never have achieved a PhD, which she found "an invaluable experience".

Her thesis research at the University of Canterbury in Christchurch estimated the results of treatment through causal effect on survival probabilities for groups of patients in randomised clinical trials, by creating potential outcome survival models with a finite number of parameters.

Estimating causal effect is complicated in clinical trials because patients don't always take treatment as prescribed (called non-compliance), and because some outcomes are missing. For example, a trial may attempt to find out the number of years patients live after receiving a particular treatment (called their survival time). If some patients are still alive at the end of a trial's observation period, this outcome is not available and their survival time is said to be censored.

Gong's PhD applied new survival models, to take into account non-compliance, time-to-event outcomes, and censoring mechanisms in randomised trials with control groups. She applied these models on data from the 1960s New York Health Insurance Plan breast cancer screening trial. "Most models ignore censoring mechanisms when making their survival models," she says. "If we ignored the censored mechanism from the model, we would have got very strange results in some situations."

During her PhD, she also took part in a study of the variability in hospital treatment outcomes with other staff at the Department of Public Health and General Practice. The study found almost a two-fold variation in several health outcomes between and within hospitals, suggesting that a "hospital effect" on patient outcomes may operate at the level of wards and services.

Gong also created computer programmes and analysed clinical research data using the Statistics Analysis System software as an assistant research fellow in the department. She has since had a bout of serious illness, but is applying for jobs in New Zealand working in health statistics. "I would like to analyse data on a range of health topics," she says.

$$S(t) = Pr(T > t)$$

Photo: Keri Moyle, www.signsoflife.co.nz

Better cycling networks

Andrea Raith, right, now a lecturer at the University of Auckland, is at the leading edge of traffic problem solving.

She started by developing solution methods for the design of wind turbines in Germany as part of her studies at the Technische Universität Darmstadt, near Frankfurt. "I devised bi-criterion optimization solutions for a new type of electricity generator in wind turbines. It wasn't off-the-shelf – it had to be calibrated for each location. We had to determine the parameters - size, diameter, number of magnets in the generators – so we could optimize efficiency and weight, which are conflicting objectives." She then extended the method to deal with more than two objectives, and it was used by a major wind power company.

She said she couldn't have done her PhD on transportation optimization problems at the University of Auckland without an NZIMA scholarship.

She applied bi- and multi-criterion optimization to three different problems. She studied the bi-objective version of the widely-known shortest path road network problem, which "included the safety and suitability of the route. It was fairly theoretical, comparing different algorithms and trying to improve them."

The second was the network flow problem about moving commodities, for example logistics networks through which goods are transported; "there wasn't an algorithm so we proposed one". The third problem was traffic assignment, for example road networks in rush hour, when everyone wants to get to work as quickly as possible. "Planners use these algorithms to see the impact of a new road on traffic patterns."

"They usually assume that people travel on the shortest route, and use a generalized cost function that converts time into cost and combines it with other vehicle costs. But it's not realistic - people choose different routes because of convenience or different objectives. With a toll road, for example, some people will use it because it's quicker, while others will always go the long way to avoid the toll. We came up with one algorithm and different ways of dealing with those objectives."

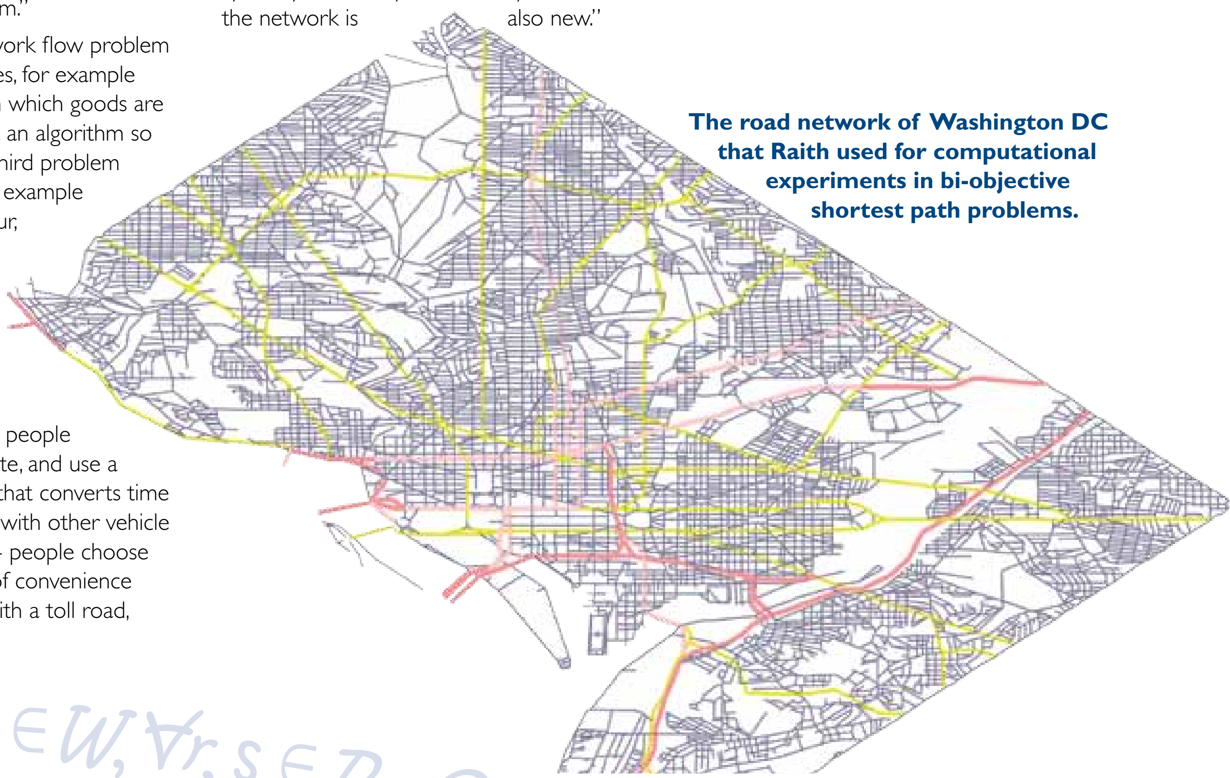
She and her two PhD supervisors, Matthias Ehrgott and Judith Wang, were awarded a Marsden grant to find better ways of solving this multi-objective traffic assignment problem.

She is involved in a project applying a similar theory to Auckland's cycling network. "Cyclists want to minimize their time and maximize the suitability of the path - this includes 20 factors, such as road gradient, traffic volume and speed, road surface and width, combined into a score."

She found that new cycling paths are often decided on in isolation, rather than on how they fit into routes that people want to follow. "We thought it was important to capture that. The route choice idea is new for cycling, and integrating an assignment approach that determines how many cyclists you can expect on every road in the network is also new."

However, convincing, or even contacting the Council decision-makers isn't so easy.

She's also using optimization to manage water assets and solve factory production problems. "That's the interesting part of the job - you can link with the real world and help them solve problems."



The road network of Washington DC that Raith used for computational experiments in bi-objective shortest path problems.

$$\forall w \in W, \forall r, s \in R_w \; C_{\cdot s}(f^*) \geq C_{\cdot r}(f^*) \Rightarrow f^*_s = 0$$

Finding the underlying patterns

Former NZIMA PhD scholarship holder Dion O'Neale enjoys seeing the same mathematical patterns in completely unrelated fields.

O'Neale has applied mathematical modelling methods to fields as different as astronomy (Saturn's rings), electricity generation (geothermal reservoirs), physics (laser light oscillation), chemistry (nano crystals), and economics (research investment).

He didn't expect to specialize in mathematics, but got hooked by a university research project that used maths to solve an astronomy problem. "I looked at whether sunlight hitting the particles in Saturn's rings affects their position, and it does!" He majored in maths and physics for his BSc, and for his MSc at Düsseldorf in Germany he developed numerical integrators to solve differential equations describing relativistic interactions between lasers and charged particles.

The scholarship from the NZ Institute of Mathematics and its Applications meant "not having to use tutoring to get by at Massey, otherwise I would have taken much longer to do the PhD". He was also awarded NZIMA travel funding to attend part of the six-month programme on highly oscillatory problems at the Newton Institute in Cambridge, and the International Conference on Scientific Computation and Differential Equations in France.

Highly oscillatory problems involve light, sound or other elements that oscillate very quickly. "If you're doing a laser experiment, you want a simulation to predict what might happen over longer periods without having to worry about every little wavelength."

This background in geometrical numerical integration, Hamiltonian systems and differential equations enabled him to develop mathematical models for many different fields as a research scientist at Industrial Research Ltd. "If an electricity company wants to set up a new geothermal power station and has reports of what's under the ground, a mathematical model tells them they're likely to be able to make this much electricity for this many years before the field stops being viable, as well as how much water they have to re-inject into the ground to avoid subsidence."

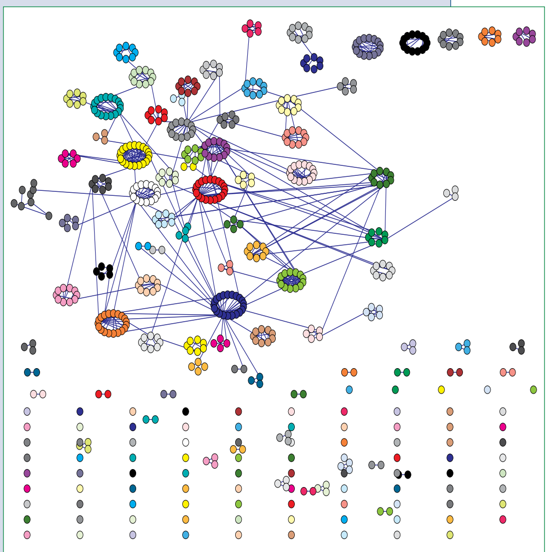
His latest project attempts to "bring some hard numbers to the innovation ecosystem". He has examined how patents are distributed between companies in New Zealand and overseas and the relationships of research and development funding to patents, as part of an IRL project led by Professor Shaun Hendy.

"We found that the model followed a power law, and could describe the relationship of the spread of patents in a population of companies with a single number. The more you spend on R&D, the higher your rate of return in patents up to a threshold of about three percent of GDP. New Zealand only spends about 1.3 percent, so there's plenty of room for improvement."

Government innovation policies and the level of collaboration among scientists also affect the rate of return on R&D. The major difference between New Zealand and other OECD countries is the low investment by New Zealand businesses – they contribute 0.5% of GDP in R&D investment, 38 percent of the total, compared to two-thirds of the total in other OECD countries.

O'Neale borrowed this model from biology, where it described the spread of species in different genera. "I like being able to take the same piece of computer programme and use it in an entirely unrelated area; seeing the underlying patterns in things, looking a little deeper into the world than you could without the mathematics."

See also: http://frontlawn.blogspot.co.nz/



A detail from a patent network for some companies in Denmark; companies are dots and lines are shared patents or inventions.

O'Neale rock climbing at Mt Arapiles in Victoria in 2009. Photo: Chris Tuffley.

Tracing the body electric

Andrew Pullan was an outstanding bioengineering scientist who died in March 2012 in the middle of a stellar research and teaching career in the Auckland Bioengineering Institute and the Department of Engineering Science (DES) at the University of Auckland.

Pullan grew up in South Auckland and was dux of Aorere College, where he later donated (and regularly presented) the Pullan Cup for Dux. His research ranged from theoretical aspects of the microscopic function of stomach cells, to computer models of the electrical function of the heart, gastro-intestinal system and other body areas, to developing measuring devices and analysis techniques for improved health care.

Says a friend Professor Ron Paterson: "He was fascinated with the human body and loved thinking about anatomy and physiology". After a PhD modelling groundwater, he found his career focus when he started modelling electrocardiography. Says Professor Martyn Nash: "The standard 12-lead ECG provides sparse measurements, and doctors largely use pattern recognition and educated guesswork to infer what is wrong with the heart."

Pullan and his team designed a vest with several hundred ECG electrodes, and developed numerical methods to infer heart health from this dense array of surface signals. For these inverse problems, he solved partial differential equations using detailed computer models to investigate the relationship between the two. He attracted large research grants, teams and collaborators in three universities in the USA, eventually writing a textbook on mathematical modelling of the heart. Pullan become a professor in 2006 and was elected a Fellow of the Royal Society of New Zealand in 2009.

He used the same ideas to model skeletal and smooth muscle, more recently in the gastrointestinal system. Our digestion operates on small electrical signals - slow waves - that push the food through the stomach and intestines. Signal abnormalities can create chronic indigestion, reflux and other problems. Members of Pullan's team designed electrodes that could be inserted down the throat to measure signals directly from the stomach, and are working out how to record them.

Pullan saw this as leading to the development of pacemaker devices for the stomach. This work was recognised with a James Cook Research Fellowship in 2003. His team of 14, now led by Dr Leo Cheng, is well-established, with different projects building on aspects of gastrointestinal function that he initiated.

Pullan was hugely supportive of his students; Associate Professor Rosalind Archer says "He had a knack of knowing when someone could do better or extend themselves". He was also known as an inspiring teacher, full of boundless and infectious energy. He opened his inaugural professorial public lecture in fluent Māori verses which he had spent a month memorising, and a large monitor displayed the electrical activity of his body in real time as he paced.

He was also famous for his shorts and polo shirt dress code, at Auckland Grammar School Board meetings and university lectures, and for his intense competitiveness in any physical activity. He established the highly successful Next Top Engineering Scientist problem-solving competition for secondary school students in 2009, which now offers the Pullan Prize, and the DES Christmas lunch for families and children. His love of chocolate meant he was a regular judge of the annual student maths-themed baking competition, and many of his postgraduate students have vivid memories of their labour on his extensive house renovations.

Says colleague Professor Bruce Smaill: "He was fearsomely bright, extraordinarily energetic, and he cared intensely about everything that he did."

$$\nabla(\sigma e \nabla \phi) = \nabla . J$$

See also

The Next Top Engineering Scientist competition - www.des.auckland.ac.nz/uoa/home/for/secondarystudentsandschools/nztescompetition

Mathematics and the human stomach - www.math.auckland.ac.nz/CULMS/wp-content/uploads/2010/08/Maths-and-the-Human-Stomach.pdf