NZ IMAges

ISSUE FIVE

New Zealand Institute of Mathematics & its Applications

Modelling and heart patients

Staff at the Cardio-thoracic and Vascular Intensive Care Unit (ICU) of Auckland City Hospital have found a mathematical model of the unit's operation valuable in improving its efficiency. Jenny Rankine reports.

Unit clinical director Dr Andrew McKee says juggling staffing, theatre availability and beds is complex; "it's hard to match the resources to the demand".

A chance conference meeting in 2006 started a collaboration between statistician Ilze Ziedins, pictured, BSc (Hons) and Masters student William Chen, and unit staff on a queuing model that could simulate the effects of operational changes on patient numbers. Associate Professor Ross Ihaka co-supervised Chen's Master's thesis, and advised on constructing the simulation and other aspects of the project.

At the time, the unit admitted around 22 patients a week, some for elective surgery and others with acute problems needing intensive care. "The bottleneck was the intensive care unit," says Ziedins. "Around half the patients stay for a day or less, some stay for much longer. We modelled the flow of scheduled elective and other patients into the unit, with random acute arrivals and lengths of stay, simulating 24 hours and seven days a week."

The model gradually became more complicated, taking into account the cluster of arrivals around midday and after 4pm after surgery, variations by day of the week and different kinds of patients. "Since arrival rates change over the duration of a patient's stay, traditional queueing models are not helpful, and new analytical models will need to be developed," says Ziedins.

Chen wrote the simulation programme from scratch using the statistical software R (see IMAges 3). Each simulation run was for a year of the model's operation, and this was repeated several times to obtain confidence intervals for measures such as the average number of cancellations. The initial aims were to reduce waiting times, and cancellations of elective surgery due to the arrival of people with acute problems. "The model demonstrated that we needed more staffed ICU beds to match operating theatre capacity," says McKee. "We had an average of nine and needed 12 to manage our expected number of patients."

Having an external analysis independent of clinical pressures was a powerful argument for more staff, he says. The unit now has a higher allocated staffing level, although the international shortage of clinical staff has meant not all the positions have been filled.

The aim then shifted to matching the nursing roster with the patient load. "We're working on that now using a stochastic optimisation model," says Ziedins. As rosters are done three months in advance, the evaluation cannot start until the current roster ends in mid-November. "We think improvements can be made; they may be able to treat one or two more patients a week, which is substantial over a year."

"The whole unit has been very interested," says Ziedins. "Up to 20 people have turned up for

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Welcome

We hope you enjoy this, our fifth issue of IMAges, which contains a range of items about the work and interests of the New Zealand mathematical community, plus an item on the mathematics of origami as well as photos of a recent event where schoolchildren were invited to participate in the creation of a large Penrose Tiling. Read about this and more inside.

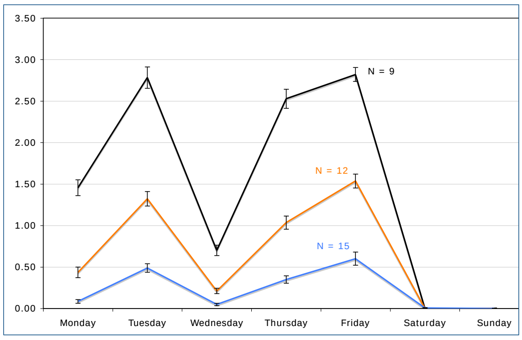
Marston Conder and Vaughan Jones
Co-Directors

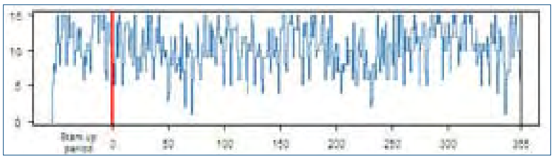
presentations; the input from them is wonderful."

"We can use the model to analyse our patterns of work," says McKee. "For example, we can see if it would make any difference to patient throughput if we could discharge all the patients back to the ward an hour earlier after surgery."

Queuing theory is often used to analyse phone, internet and road networks, as well as customer services such as banking. "Once you think about life, almost everything starts to look like a queue," says Ziedins. It's the randomness that's important. Reducing variability can have a marked effect on how systems perform. For example, lights on Auckland motorway on ramps reduce clumping and make traffic flow less congested."

"I find this project so rewarding because it has been an opportunity to make a difference - some people might receive treatment earlier as a result," she says.





Above: The average number of ICU cancellations per day, with different numbers of staffed beds. This plot was produced in the project's first year, when the model assumed a constant number of staffed beds during the week and not all patients were included. Only one operation session is scheduled on Wednesday, leaving time for meetings and training.
Far left: William Chen. Left: The number out of 15 beds occupied overnight for a single simulation run over a year.

$$e^{-\lambda/\mu}\frac{(\lambda/\mu)^n}{n!}$$

> In how many other disciplines is there anything that parallels the statement that Gauss's formula for triangular numbers will ***never*** fail to give the right answer?
>
> *Marcus de Sautoy, Music of the Primes.*

NOTABLE MATHS PROBLEMS

COLLATZ CONJECTURE

Take any positive integer; if even, divide by two; if odd, triple it and add one. Using each result as the input for the next step, no matter what number starts the sequence, the end result is 1.

Simply: That all paths in a certain kind of number sequence eventually lead to 1.

Also known as: The $3n + 1$ conjecture, the Ulam conjecture (after Polish mathematician Stanislaw Ulam), the Syracuse problem or HOTPO (Half Or Triple Plus One) in computer programming.

Discipline: Number theory.

Originator: German mathematician Lothar Collatz, 1937; he made very little progress and published a history of its origin much later.

Incentive: $US50 by H Coxeter, 1970; then $US500 by Hungarian mathematician Paul Erdős; £1,000 by B Thwaites, 1996; solving a problem with a tantalizingly elementary form that has eluded top mathematicians.

Examples: If $n = 6$, the number sequence is 6, 3, 10, 5, 16, 8, 4, 2, 1. If $n = 27$, the sequence takes 111 steps, climbing to over 9,000 before descending to 1.

Explorations: Many attempts have been made to settle the conjecture using technologies from number theory, dynamical systems and Markov chains. USA mathematician Jeffrey Lagarias proved in 1985 that the problem has no nontrivial cycles of length <275,000. Another approach took the opposite direction; instead of proving that all natural numbers eventually lead to 1, it proved that 1 leads to all natural numbers. USA mathematician John Conway proved in 1972 that Collatz-type problems can be formally undecidable.

State of play: Although the conjecture has not been proved, most mathematicians who have worked on the problem believe it is true. The conjecture has been checked by computer for all starting values up to 10×2^{58}. However, some important conjectures have been found to be false only with very large counterexamples.

Each odd number in Collatz sequences is on average ¾ of the previous one, leading to an argument that every Collatz sequence should decrease in the long run. This is also not a proof because it pretends that the sequences are assembled from uncorrelated probabilistic events.

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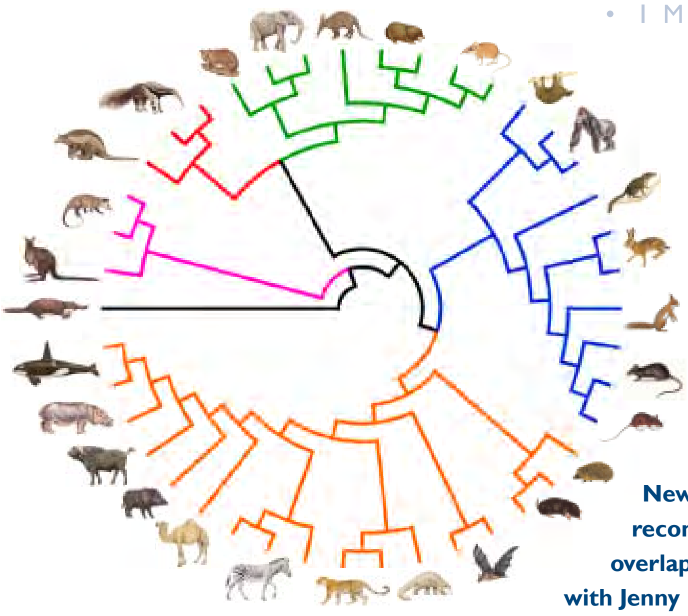


Recipes for supertrees

New Zealand is at the international forefront of phylogenetic reconstruction algorithms, which attempt to build a genetic data set from overlapping species into a single evolutionary tree. Charles Semple spoke with Jenny Rankine.

Creating a best-fit evolutionary tree for 20 species is extremely difficult, yet biologists now regularly try to build supertrees of hundreds or thousands of species. There are often conflicts in the genetic data. Sometimes these are errors, but they may represent the combination of different species into a new hybrid, lateral gene transfers or recombinations of DNA strands.

"These events are more common than people first thought," says Associate Professor Charles Semple, who is based at the University of Canterbury. "Sometimes two genes simply evolve in different ways." In these cases, instead of seeking a single tree, mathematicians can create a network that highlights the conflicting genetic signals.

Semple is a co-director of the NZIMA programme in algorithmics with Professor Mike Atkinson at the University of Otago and associate director Dr Mark Wilson at the University of Auckland. The programme aims to link local and overseas algorithmic researchers, and raise awareness of the field with high school, undergraduate and postgraduate students.

An algorithm is a list of well-defined instructions for completing a task from an initial state through well-defined successive states to an end-state. "A cake recipe is an algorithm," says Semple. "You have to do things in a certain order otherwise the cake may sink in the middle or fail to rise."

Computer games are a more common example. Algorithms are also behind packet routing on the Internet. Another application is the use of phylogenetic algorithms by Russell Gray's group at the University of Auckland to analyse the development of Pacific languages. This has challenged existing theories about Pacific settlement patterns.

Many problems are so intractable that even the best algorithms may be unsolvable with a supercomputer, or take days to run. Semple gives the example of timetabling problems. "Universities need to schedule a lot of exams in a very short time, but want to avoid students having to attend two exams at the same time. The best solution is trial and error, but there are so many possibilities it can take a computer a long time to run through them." Semple will use examples like these as part of the programme's outreach to high school students in term four.

The programme's first event, a Workshop on Algorithms in Napier in February, enabled programme Masters student Josh Collins to reduce computer time on a problem from two days to two hours.

Algorithms are playing the role for many sciences that formulae play for physics, says Semple. "While faster computers enable far more extensive use of algorithms than we imagined a decade ago, the great leaps forward come from better algorithms, not better hardware."

Above: A phylogenetic tree of 42 representative mammals reconstructed by Frédéric Delsuc and Nicolas Lartillot using a Bayesian approach, concatenating 12 protein coding mitochondrial genes.

Below: Participants in the Workshop on Algorithmics in Napier, February 2008.

while, if, else, input, compute..

Nano-maths

Mathematics and nanotechnology are ideal partners, according to Dr Shaun Hendy of Victoria University and IRL. Anna Meyer explains.

What happens to physics when things get really small? How can we investigate structures so tiny that each individual atom is important? And how can useful nano-devices be built? Mathematics and computer simulation are helping to answer some of these fundamental nanotechnology questions.

"Using computer simulation, we can study materials right down at the nanoscale," Dr Shaun Hendy explains. "We might be interested in a nano-device containing millions of atoms; we can use the simulation to visualise how each of these atoms behaves." Part of the advantage of this 'silicon-laboratory' approach is the ability to see exactly what is going on at incredibly small scales, which is normally very difficult, expensive and time-consuming.

The computer simulations complement laboratory work by other research teams. "I work with several experimental groups in the MacDiarmid Institute for Advanced Materials and Nanotechnology, including Simon Brown's at the University of Canterbury."

"Simon's team builds nano-electronic devices using very small particles called nanoclusters. We're doing a lot of the modelling behind the manufacturing, to help figure out new and better ways of doing it. Some of the work we've done has led to new patents for his spin-off company."

Dr Hendy's team is one of the biggest computing consumers in New Zealand, using the University of Canterbury's Blue Gene supercomputer, the most powerful in the southern hemisphere. But even this is not always enough for the types of modelling the team is doing.

"We could use all the computing resources in the world and it would not be enough," said Dr Hendy, "so we're always trying to come up with clever ways of using the computer, of re-casting the mathematical problem in a slightly different way."

The team also constructs more traditional mathematical models of events at the nanoscale. In the world of the very small, even physics itself is different, and this can be used to make new types of devices. For example, surface tension becomes very important, as objects at that scale have very high surface area to volume ratios.

"We have had a lot of fun just constructing mathematical models of quite common phenomena, such as melting, and then seeing what happens when surface tension takes over," said Dr Hendy. "It's surprising how your intuition, which is tuned for a human-sized world, can get things completely wrong when you try to guess what will happen at the nanoscale."

Shaun Hendy, left, and Dmitri Schebarchov.

The group also work on what are known as homogenisation problems. "These are similar to what happens when you put batts between the wooden beams in your roof, creating a heterogeneous or mixed up layer of insulation in your ceiling. You may want to know how well this mixture of batts and beams insulates your roof on average."

"A similar situation occurs at the nanoscale, where atoms of different types are arranged in patterns. We are interested in how these patterns of atoms lead to the overall properties of a nanomaterial." NZIMA-funded postdoctoral fellow Dr Philip Zhang and PhD student Nat Lund have been active in this area.

Another focus is nanofluidic devices – tiny pipes already used for applications such as rapid DNA sequencing. As the pipes are made smaller, the pressure needed to push fluids through them increases dramatically. "People make these lovely little devices, but then the pump needed to run them is the size of a table – it's very embarrassing. Again, the problem is surface tension – the drag on the fluid increases as you make the channel smaller. We are trying to help reduce this problem."

Finally, NZIMA-funded PhD student Dmitri Schebarchov has been working on understanding the details of very small capillaries, such as carbon nanotubes. He and Dr Hendy have discovered a way to fill and unfill them with metals, which has not been achieved before. "This will be important for building different types of nanostructures," explained Dr Hendy, "and also has relevance to controlling nanotube growth, which can be difficult."

"There's a lot of fun to be had, and it is a good place for mathematical modellers as there are a lot of new mathematical problems to be found. The work is a mix of physics and mathematics – I like the challenge of solving mathematical problems for their own sake, but I also like the fact that there are applications at the end of it. It's also a field that is developing really rapidly. You certainly don't get bored - there's always something new that you can work on where you can make progress."

Secret keys and colluders

$$(a_{i,j}) = \left\{\begin{matrix}1001\\0111\\1001\\1100\\1101\\1111\end{matrix}\right\}$$

Anyone trying to keep track of their passwords for work, email, internet banking and other websites will understand the trade-off between security and efficiency. They have to remember more and more secret keys, which are secure unless one is forgotten, or they use the same key for every site, which is efficient but not secure. By Jenny Rankine.

Julia Novak is exploring the pure maths of Key Distribution Patterns - a method of reducing the amount of secret information that needs to be stored for secure communication between large networks of users. This could apply to any internet-based application, communication within large corporations, or in agencies where secret information needs to be protected such as the military.

While she is lecturing at the University of Auckland, and occasionally working for the NZIMA, her PhD in combinatorics is being supervised from Royal Holloway at the University of London. She is using incidence structures and block designs from design theory. Designs have points which can be associated with network users, while blocks are associated with security keys.

"There is always a trade off in secure communication networks between efficiency and security," Novak says. "Efficiency is a measure of how much secret information has to be produced, distributed and stored securely, and security can be measured by the minimum number of parties who share their secret keys - called colluders - that will break down the system's security.

All systems attempt to use as few unique keys as possible, while maximising the number of colluders needed to crack security. "A system is called x secure if x colluders will not be able to access anyone else's secure information."

Common systems use a mix of published and secret information. While keys remain secret, reference numbers for patterns of users to keys are published. "A one-way function is also published. It takes several keys as an input and outputs a digit key for each group of users who are trying to communicate securely."

Novak says the maths is "all about uniqueness and commonality". She specifies a group G, made up of families of privileged users, and a group F, made up of families of forbidden users, so that even if all members of F share information, they cannot access the keys of the privileged users.

None of the previous maths had taken into account the roles of individuals. For example, people who are less trustworthy may be put in group F, while people at the top of an organisation may not appear in any F family.

"This means setting upper and lower bounds. For example, what is the maximum number of keys users have to hold for a Group Key Distribution Pattern (GF-KDP) to work?" Usually the organisation contracting a secure system will specify at least one boundary.

These systems can be represented as binary matrices, with users as rows and keys as columns; a user with a key is represented by 1 and one without by 0. "For any binary matrix that meets certain conditions, I can read off a maximum set of privileged families. After I get that, I can find the maximum set of forbidden families for that set of privileged families. If the maximum set of forbidden families is specified first, then this restricts the maximum set of privileged families. From any one pattern of keys to users, you can have higher security and fewer secure communications, or more secure communications and lower security."

While the maths is still theoretical, it could be added to existing cryptographic security systems to improve their efficiency.

Awards and honours

BILL BARTON, director of the new NZIMA programme in Mathematics Education, has been elected the next President of the International Commission on Mathematical Instruction (ICMI), from 2010 to 2012.

NZIMA Co-Director **MARSTON CONDER** became the Royal Society of New Zealand's first Vice President International when his term as President of the RSNZ Academy ended on June 30 2008.

Professor **MIKE EASTWOOD**, one of the NZIMA 2008 Visiting Maclaurin Fellows, has won a Federation Fellowship from the Australian Research Council.

GAVEN MARTIN (one of the NZIMA principal investigators and co-director of the programme on Conformal Geometry) has been awarded the Hector Medal for 2008 in mathematical and information sciences, by the Royal Society of New Zealand.

NIC SMITH, director of one of the first NZIMA programmes (on modelling cellular function), has been invited to co-direct a programme on the Cardiac Physiome Project: Mathematical and Computational Foundations at the Isaac Newton Institute in Cambridge, UK from June to August 2009.





Infinity without a repeat

The Penrose Patterns activity during Incredible Science day at the University of Auckland in July involved 2,000 kite and arrowhead shaped perspex tiles in five colours.

With these two simple shapes, primary and intermediate students could create kings and queens, suns, stars, birds and worms, and learn why some patterns could go on infinitely without ever repeating.

Hugh Gribben had the idea and got the tiles made. Each tile had a hole in one corner and there were two simple rules - the corners with holes had to go next to another corner with a hole, and the long edges had to go next to other long edges. Hundreds of students took part, most starting their own small patterns separately all over the room. “Kids who could hardly speak could put tiles together,” said Dr Isabel Hubard, who ran the project. “But at one point we got stuck, because it can be impossible to join lots of small groups of tiles.” Hubard changed the colours on one section with lots of symmetries and told the children how to resolve the problem. “Then it was big enough that people could add tiles all around the edge and not get stuck.” Photos: Tiger Tiger.







Playing with polytopes

Like many mathematicians working with symmetries, Dr Isabel Hubard started working with polyhedra because she loved their beautiful shapes. "I like to draw pictures, use different colours and study them for hours. Playing like a five-year-old, I can see which mathematical properties they satisfy." Hubard prints out her own triangular and hexagonal graph paper for doodling.

Hubard, who hails from Mexico City, touched down in Aotearoa for 2008, after a Masters and PhD in York University in Toronto, and before a post-doctoral position in Brussels, Belgium. She is a temporary lecturer at the University of Auckland in graph theory.

Mathematics orders the relationships of vertices to edges and faces in polyhedra. "A vertex is smaller than an edge if it is one of the endpoints; an edge is smaller than a face if it belongs to the face. Then we can forget about what they look like physically and think of them as objects with an order between them. We can use as many layers or ranks as we like. We say the faces are of rank 2 to any finite number; they are abstract polytopes, unable to be built in the 3-space we occupy."

Regular polyhedra - such as tetrahedra, cubes and octahedra - are the most studied; they include many symmetrical reflections and rotations. Other polytopes that have all the possible rotations, but not reflections, are called chiral. A related vertex, edge and face on all polytopes is called a flag.

"When polytopes are regular, the number of symmetries they have is the same as the number of flags," says Hubard. "And the reverse is true for finite polytopes – if the number of flags is the same as the symmetries, then the polytope is regular."

"For finite chiral polytopes, the number of symmetries is half the number of flags. But the reverse doesn't work - if you have half the symmetries, it doesn't mean that it's chiral." Hubard studied two-orbit polytopes for her PhD. Symmetries can divide the flags into two sets, called orbits. "Given a two-orbit polyhedron, we can tell the group of symmetries, with its generators and some relations they will satisfy. If you give me any symmetrical group, with generators and relations, I can tell you if it will be a group of two-orbit polyhedra and if it is, I can construct it."

She also worked on what happens to polyhedra when the order of vertices, edges and faces for each shape is reversed. "When we reverse the order, we get a different polytope. A cube becomes an octahedron and a dodecahedron becomes an iscosahedron. But a tetrahedron becomes a tetrahedron in a different place; this is called self-dual."

"It is intuitive to think that if we reverse the order twice, we get the same object. The problem is that the same object may end up in a different position." After two reversals, regular polytopes can always to return to the same position; those with very little symmetry don't always return to the same position. Hubard and her supervisor Asia Weiss found that chiral polytopes of odd ranks can always come back to their initial position, but that some of those of rank 4 cannot. Later, with Alen OrbaniĐ, Hubard and Weiss found a rule that decided for any polytope.

She is now working on generalising from two-orbit polytopes to those of any rank, with Professor Egon Schulte at North Eastern University in Boston. And when she's not doing maths, she's teaching tango in downtown Auckland.

MATHEMATICAL EVENTS

24-25 November, Wellington
Operations Research Society of NZ 43rd Annual Conference
http://conference.orsnz.org.nz

7 December, Christchurch
Half-day Workshop for Women Researchers in the Mathematical Sciences in NZ
Contact Vivien Kirk, v.kirk@auckland.ac.nz

8-12 December, Christchurch
7th Australia-NZ Mathematics Convention (incorporating the 2008 NZ Mathematics Colloquium)
www.math.canterbury.ac.nz/ANZMC2008

15-19 December, Auckland
4th International Conference on Combinatorial Mathematics and Combinatorial Computing
www.cs.auckland.ac.nz/research/groups/theory/4ICC/index.html

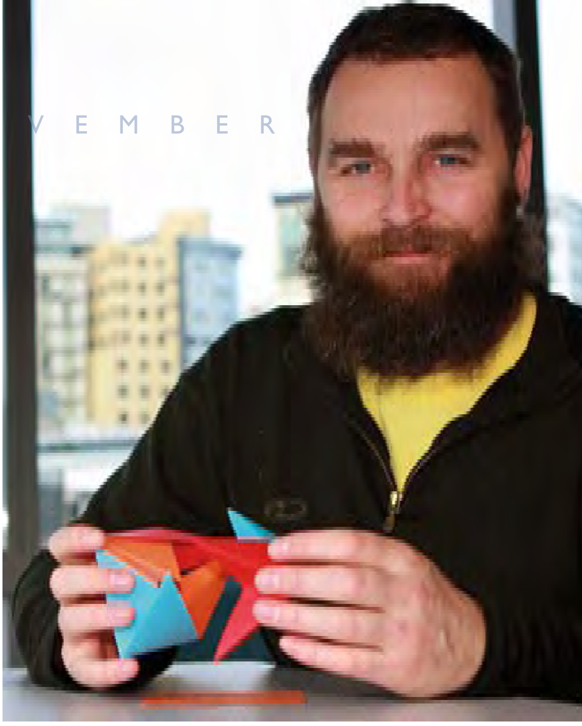
4-9 January 2009, Napier
Annual NZMRI/NZIMA Summer Meeting, on algorithmic information theory, computability and complexity
www.mcs.vuw.ac.nz/Events/NZMRI2009/WebHome

1-5 February, Caloundra, Queensland
ANZIAM 2009 (annual meeting of Australia & NZ Applied Mathematics)
www.sci.usq.edu.au/conference/index.php/ANZIAM/2009

6-10 July, Sydney, Australia
First Pacific Rim Mathematical Congress
www.primath.org/prima2009/

29 September - 2 October, Palmerston North
Pi in the Sky: Extending Mathematical Horizons, Biennial Conference of the NZ Association of Mathematics Teachers (NZAMT11)
www.nzamt.org.nz/nzamt11/

Learning by folding



Don't be surprised to see an origami workshop advertised at a high school near you in 2009. New Zealand origami master Jonathan Baxter and Auckland mathematician Hugh Gribben, pictured, took their Great Origami Maths and Science Show on a highly popular national tour in 2006, and they're keen to repeat it. By Jenny Rankine.

Gribben folds coloured pieces of paper as he speaks, creating elegant shapes or modular polyhedra. But folding and talking don't go well together unless he practises, and after a conversation his desk is littered with finished and half-finished shapes, intriguingly creased.

He describes the tour as a "whizz-bang show, not a workshop", which used origami to turn kids onto maths and their teachers onto its ability to illustrate tangibly many maths areas. The show also explored science principles, such as aerodynamics with paper darts and how DNA helixes collapse.

Students were bussed in to main centres for the show, which had booked out audiences of up to 300. The pair folded on stage, with live close ups on a big screen, and invited some students on stage to fold particular shapes. Each class received a copy of the 150-page book of the show.

Gribben has used origami as a first year maths guest lecturer, and is regularly invited to run school workshops. "It is tangible rather than abstract learning; the kids learn it with their hands." He may start by asking them how many ways they can fold a square of paper into a new shape that is half the area of the original square.

His favourite example was first documented in 1893 by T. Sundara Row and starts with all four corners meeting in the middle of the square. The new square is half the area of the original. In Row's book, this is repeated until the square is too small to continue.

Imagining the folding going on for ever provides a proof of the convergence of the geometric series. Gribben says students find this geometrical proof very convincing when they fold it themselves.

Another exercise can involve making a nautilus shell similar to the NZIMA logo, or a self-similar wave. "If I did that with 100 steps, they would all have the same ratio between the folds of the wave front." Year 13 maths students can then use complex numbers to find the centre of the spiral.

He also gets students exploring Platonic or Archimedean solids. Modular polyhedra are his favourite - "they're simple shapes put together - lots of eye candy". Gribben is pictured making an alpha prism. This semi-regular solid is made of six modules, each a paper square folded three times into a right isosceles triangular shape.

He likes to make them with three paper colours. "I make it so different colours don't lie next to each other, don't share a common edge and the outside is made up of four small triangles. These are aesthetic decisions, but they're also strongly mathematical. To make two the same is difficult - it teaches precision of thinking. It's a very simple but deeply meaningful shape."

Another exercise involves conic sections. "If students mark a point near the bottom of an A4 portrait page and fold different points of the bottom edge to that point, they get a family of folds that are all tangents to a parabola. They can do similar things with an ellipse and a hyperbola."

Famous ancient problems such as doubling a cube and trisecting an arbitrary angle, which the Greeks attempted unsuccessfully to solve with compass and straight edge, can be solved with origami. "I like to believe I could come up with a workshop about any mathematical area," says Gribben.

Computational origami has solved engineering and science problems, such as folding and opening space telescopes and solar panels on satellites. "Studying paper crumpling is an easy way to learn how to model bumpers crumpling during car crashes, or plane bodies on impact," he says.

"Airbags are another application. They have to unfold very rapidly without hard edges." The solution used algorithms developed by Robert Lang for his Treemaker origami programme. "A stent, which has to move along blood vessels then open up and lock into place, uses a very simple origami technology."

Once Gribben has finished maths studies for a Postgraduate Diploma in Science he wants to be "an itinerant maths teacher, running workshops, doing tangible learning". Watch out for Gribben and Baxter in 2009.

Top: A folded base, and the finished scorpion folded from that base.

Background: Screen shot of the computed crease pattern for the scorpion using TreeMaker 4. Circles correspond to terminal flaps.

See also

The Great Origami Maths and Science Show book, available for $27 from Origami New Zealand, c/- Rotorua Arts Village Experience, 1240 Hinemaru St, Rotorua, phone 07 348 9008, fax 07 343 7108, email jbax@mindspring.com

The GOMSS website - www.nzamt.org.nz/origami.htm

A talk by Robert Lang - www.ted.com/index.php/talks/robert_lang_folds_way_new_origami.html

Robert Lang's website - www.langorigami.com/science/science.php4